Name: _____KEY_________ Email: _________________________ MARS 6920_Spring 2010

Homework Set #3 (10 points)

1) The simplest formulation of density-dependent equation does not include a carrying capacity term (K). Rather, the growth of this population responds to the numbers of individuals in the previous time step (this scenario relies on discrete time steps):

N(t+1) = Lambda * N(t) * (1 – b N(t))

Where: Mean Lambda = 3.1 and 95% CI Lambda = 2.1 – 4.6

Using b = 0.019 for all the calculations, check out the implications of the variability in the Lambda values (the mean point estimate, the lower 95% CI, and the upper 95% CI) in the expected behavior of the population.

Leaving b fixed at 0.019, do the following three times (for Lambda values of 2.1, 3.1, and 4.6).

First: Calculate the equilibrium population size (where N(t+1) = N(t)). Show me your calculations and the results of the three equilibrium population sizes (Ne). Hint, replace N(t+1) and N(t) with Ne in the equation above and solve the math after plugging in the Lambda and b.

Note: Based on the definition of equilibrium: N(t) = N(t+1) = Ne

(manipulating the equation correctly to get Ne: 0.5 points)

Ne = Lambda * Ne * (1 – b Ne) (multiply out the terms)

1 = Lambda – (Lambda * b * Ne) (divide both sides by Ne)

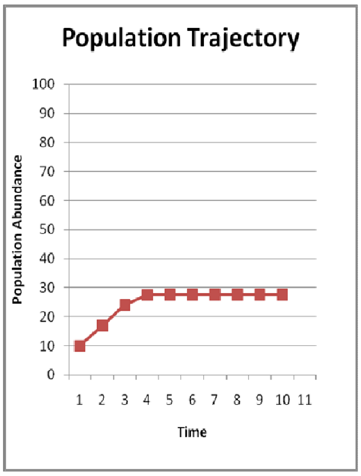
Ne = (1 - Lambda) / (Lambda * b)

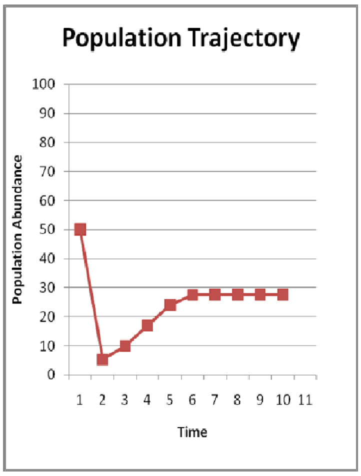
- lambda: 2.1 Ne: 27.56 (0.25 points)
- lambda : 3.1 Ne: 35.65 (0.25 points)
- lambda : 4.6 Ne: 41.19 (0.25 points)

Second: Use the excel sheet "question 1" to explore the behavior of this population. For three scenarios (refer to them as Lambda = 2.1, Lambda =3.1, and Lambda = 4.6), start the populations at two initial sizes (10 and 50). Make a plot of the two population trajectories (numbers of y axis, time on x axis) over the first 10 years, showing both initial population sizes (10 and 50) in the same graph. Explain how the population behaves in all three Lambda scenarios (does it reach equilibrium, does it crash, does it cycle?). If the population equilibrates, compare this equilibrium point with the Ne value you calculated above (for the same Lambda scenario).

(Figures: 0.50 points for each Lambda value)

(Explanations: 0.25 points for each Lambda value)

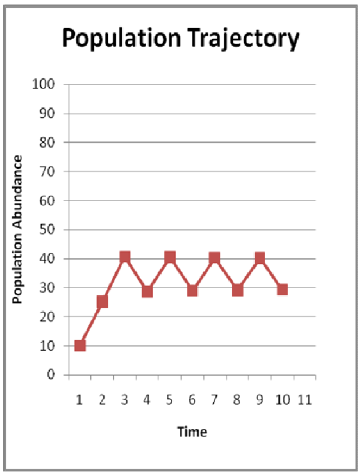


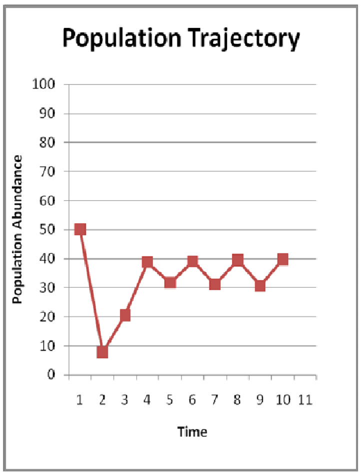


- lambda: 2.1

Population reaches equilibrium level (~ 27.56) smoothly, without overshooting or describing cycles.

The population trajectories do not vary whether we start with 10 or 50 individuals.

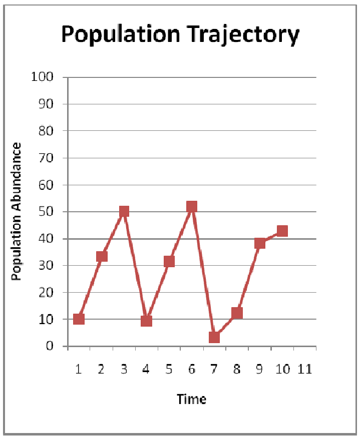


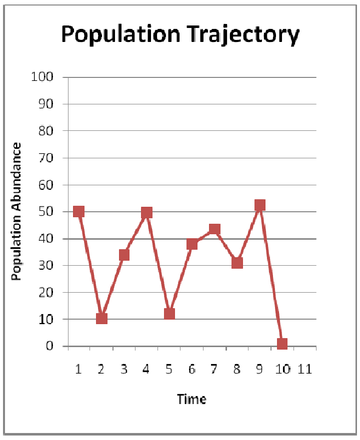


- lambda: 3.1

Population dos not reach the calculated equilibrium point (35.65), but cycles between two values (describing a two-point cycle).

The population trajectories do not vary whether we start with 10 or 50 individuals.





- lambda: 4.6

Population dos not reach the calculated equilibrium point (41.19), and does not undergo any cycles. Rather, the population undergoes wild fluctuations. Moreover, the population trajectories vary whether we start with 10 or 50 individuals.

Third: For each Lambda scenario, check out where the 1:1 line (blue line) and the recruitment curve (red line cross): This is the point of equilibrium. However, if the slope of the recruitment curve is too large (curve is too steep) at the equilibrium point, it will become unstable and the population will not stay at equilibrium. To figure out the slope, we take the derivative of the formula with respect to N(t): dN(t+1) / dN(t)= Lambda – (2* b * Lambda * N(t))

Using the b value (0.019), plug in the three lambda scenarios used in the first two parts of this question and calculate the slope at the equilibrium point. Hint, plug in the Ne values you calculated in the first part in the equation, for N(t).

Report the slopes for the three scenarios at the Ne points. Hint: the curves are decreasing, so the slopes should be negative. Interpret your results, by comparing the slopes you calculated with the results of the second part. {Note: equilibrium results when slope > -1; two-point cycles occur when -1 > slope > -2; and chaos occur when -2 > slope).

(Slopes: 0.25 points for each Lambda value)

(Explanations: 0.25 points for each Lambda value)

- lambda: 2.1 slope : -0.1 (stable; population stabilizes to K)
- lambda : 3.1 slope: -1.1 (two-point cycle; population cycles)
- lambda : 4.6 slope: -2.6 (chaos; population oscillates widely)

Homework Set #3 (10 points)

2) Using a more complex formulation of density-dependence (in excel sheet "question 2"), explore the influence of the carrying capacity (K) and the population growth rate (Lambda) on the dynamics of the population. You will use two metrics to describe the stability of this population, as you change Lambda and K: (i) the CV of the population size over 100 years, and (ii) the minimum population size (e.g., did the population go extinct) over 100 years.

   First: Starting at an initial population size of 10 and with K_good = K_bad = 100, try the following Lambda values: 1.5, 2, 2.5, 3, 3.5, 4. Describe the behavior of the population for each Lambda scenario (Hint: does the population reach equilibrium smoothly, or does it overshoot K and then reach equilibrium?, does the population undergo cycles? If so, how many values are involved in the oscillations?, does the population show chaotic behavior?)

   (Explanation of population trajectory: 0.25 points for each Lambda value)

   (Explicit description of CV & minimum pop size: 0.25 points for each Lambda value)

Lambda 1.5: Population increased and settled at equilibrium at N = 100 in generation 15 and did not overshoot K or oscillate. The CV was fairly small (16.6%), because equilibrium was reached quickly and there was little variation in population size. The minimum population size was 15, and occurred in the second generation.

Lambda 2.0: Increased and reached equilibrium at N=100 much faster (at generation 7), because Lambda was larger. Yet, the population did not overshoot or oscillate. Because the equilibrium was reached faster than with lambda 1.5, the CV was even smaller (11.5%). The minimum population size was 19 and occurred in the second generation.

Lambda 2.5: Population grew very fast and overshot K. Then the population oscillated (4 point-cycle, for two cycles), until the oscillations were dampened and the population settled at equilibrium (N=100). Despite these oscillations, the CV was even smaller (9.3%) than the two previous examples because the larger Lambda led to rapid population growth towards K. The minimum population size (at year 2) was larger than for the two previous examples (23).

Lambda 3.0: Population increased rapidly and overshot K at generation 4. It then oscillated – following a two-point cycle (48 cycles in ~ 100 generations). The small CV (9.8%) suggests that the oscillations have a small amplitude. The minimum population (28) occurred in second year.

Lambda 3.5: Population increased and displayed oscillations of alternating size (beginning with a smaller oscillation, then a larger, then a smaller oscillation). The population described 4-point cycles and the CV was substantially larger (32.9%) due to the larger oscillations). The minimum population size (33) occurred in the second year.

Lambda 4.0: This is the most interesting result: the population underwent chaotic behavior, with erratic and high amplitude oscillations. The population initially increased rapidly and overshot K by year 3. Due to these wild oscillations, the CV for this population was the largest (55%). For the first time, the minimum population did not occur in the second year, but instead occurred in year 44, due to a large population decline (a big dip). Its worth noting that this population went extinct, when the numbers dipped below 1 individual (0.7).

Second: Finally, start the population at N(t) = 10 and use a varying K (K_good = 200, K_bad = 50). Try these Lambda values (1.5 and 2), and report on the behavior of the population. Describe the behavior of the population – in comparison with what you reported in the two answers above (Hint: does the population reach equilibrium smoothly, or does it overshoot K and then reach equilibrium?, does the population undergo cycles? If so, how many values are involved in the oscillations?, does the population show chaotic behavior?)

(Explanation of population trajectory: 0.25 points for each Lambda value)

(Explicit description of CV & minimum pop size: 0.25 points for each Lambda value)

(Explicit comparison with results without varying K: 0.5 for each Lambda)

Lambda 1.5: The population does not reach the equilibrium point (N = 100). Instead, it oscillates following a 2-point cycle. This behavior is different from the previous model with lambda = 1.5 when K good=K bad, when the population settled at K (N = 100) by year 15. The CV is larger (20.3%) than in the model with K good = K bad (16.6%), indicative of the added instability due to the changing carrying capacity.

Lambda 2.0: This population does not reach equilibrium. It first increases to 50 by year 4, then decreases slightly and begins to describe a strange 4-point cycle, consisting of a large increase, then a large decreases, followed by a short increase. The CV is much higher in the current model (45%) than in the model with K good = K bad (11.5%).