Name: _____KEY_______ Email: ___________________________ MARS 6920_Spring 2010

Homework Set #2 (10 points)

1) You will use the following dataset to investigate the trend in a hypothetical protected species, censused every 10 years. The data are included in the enclosed excel sheet (Question #1).

year	number	ln(number)
1940	68	4.220
1950	40	3.689
1960	30	3.401
1970	27	3.296
1980	22	3.091
1990	17	2.833
2000	15	2.708

Your task is to determine the intrinsic rate of the decrease in this population (r) using the continuous population growth model ($Nt = No\ e^{rt}$). Hint: You will use a simple linear regression (in Excel) to calculate the parameter (r) mean value (point estimate) and the 95% intervals. Use only three significant digits for your estimates (e.g., 0.123).

- Show all of your work, including how you transform the exponential population growth model into a linear equation, and explicitly relate the requested parameters to the output from the Excel regression output.
- Answer these questions:

  a. Is the rate of decrease (r) significantly different from zero? Explain your answer. Include a plot of the best-fit regression and the regression results.

  The key to get this problem right is to convert the exponential equation into a line (y = a + bx). This requires taking the natural log, of both sides of the equation:

  Ln (Nt) = Ln(No) + rt Ln(e) (note: Ln(e) = 1)

  Ln (Nt) = Ln(No) + rt

  y = a + bx

Using the data in the "question 1" excel sheet, you can run the linear regression of Ln(N) (dependent variable) as a function of time (the independent variable). When you do that, this is the result:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.975
R Square	0.950
Adj. R Square	0.940
Standard Error	0.127
Observations	7

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	1.535	1.535	94.457	**0.0002**
Residual	5	0.081	0.016		
Total	6	1.616			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	49.445	4.746	10.418	0.000	37.244	61.645
Time (year)	-0.023	0.002	-9.719	0.000	-0.030	-0.017

The regression analysis yields these parameter estimates:

- Slope (r) = -0.023 (95% CI = -0.030 to -0.017) Population is declining, the slope CI does not overlap with a slope of 0. Also, note that the p value for the slope is statistically significant at 0.0002 ($p < 0.05$).

b. Using the three estimates you calculated for r (point estimate, upper 95% CI, lower 95% confidence interval), plug the values into the exponential population growth equation to calculate the year this population will go extinct (Hint: when N = 1, the population will be doomed, since two individuals are needed to reproduce).

To calculate the year this population will go extinct (Nt = 1), we need to re-arrange the equation as follows:

$(Nt) = No\ e^{rt}$

$1 = 68\ e^{rt}$

Ln (1/68) = t *r

- 4.219 / r = t

scenario	ln(1/68)	r	time	year
Point estimate	-4.219	-0.023	183.434	2123
upper 95% CI	-4.219	-0.017	248.176	2188
lower 95% CI	-4.219	-0.030	140.633	2081

c. Briefly discuss how you could use this range of estimates (the 95% CI) to build a safety factor into your management decisions (Hint: Check out Taylor and Wade paper).

Taylor and Wade discuss the use of a range of estimates (e.g., the 95% CI) rather than the "best" estimate (the mean or point estimate) to account for the variability in the observed data and trends. Thus, we would use the lower 95% CI to assess the earliest possible date of extinction. This would identify the worst possible case – given the uncertainty in the data- and would allow for the safest possible management actions to avert extinction by the earliest date predicted by the model.

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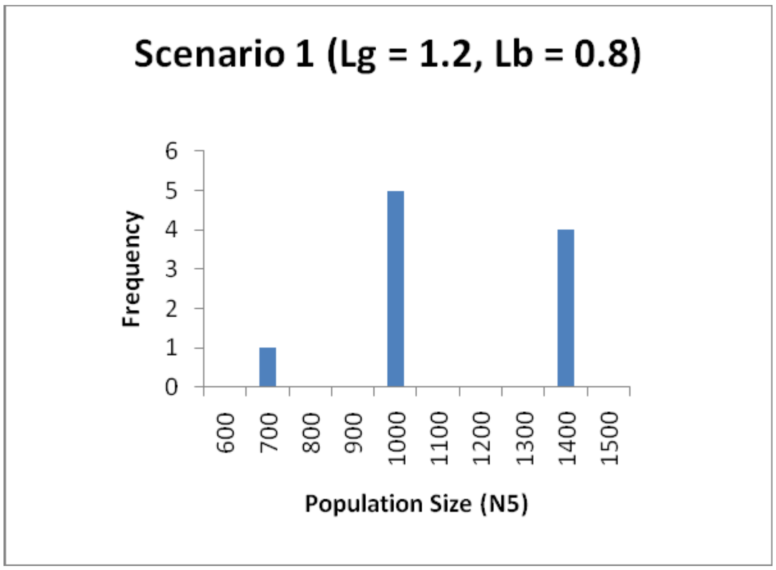
2 a) To explore the issue of random variation in the environment (stochasticity), you will use the discrete population growth equation [N(t+1) = N(t) * Lambda] to model the consequences of "good" years (Lambda = 1.2) and "bad" years (Lambda = 0.8) using the excel enclosed excel sheet (Question #2). Imagine that "good" and "bad" years occur randomly and with equal probability (p_good = p_bad = 0.5). If you start with a population of 1000 individuals at N(1), predict how large the population will be after 4 years (i.e., Calculate N(5)). Repeat this process 10 times (number of runs = 10), and provide the following:

NOTE: Because this model includes stochasticity, your specific results may vary from mine. Thus, I will explain the general patterns that we would expect, rather than providing a unique and definite answer.

The key issue is that even though p(good) = p(bad) = 0.5, when we simulate the probabilities, we can get deviations from these ideal behaviors. For instance, some runs will have four "good" years in a row, or only one "bad" year. On average, we expect half of the years to be "good" and "bad", but there are deviations. For instance, in my example, the 10 runs varied from 614.400 (3 "bad" years) to 1382.400 (three "good" years). The realized Lambda values vary accordingly, but the mean (0.973) is below parity (1).

- frequency distribution (histogram) of the expected population sizes



Note the spread of values about the expected population growth rate.

Given p(good) = p (bad) = 0.5, we would expect the population to grow at

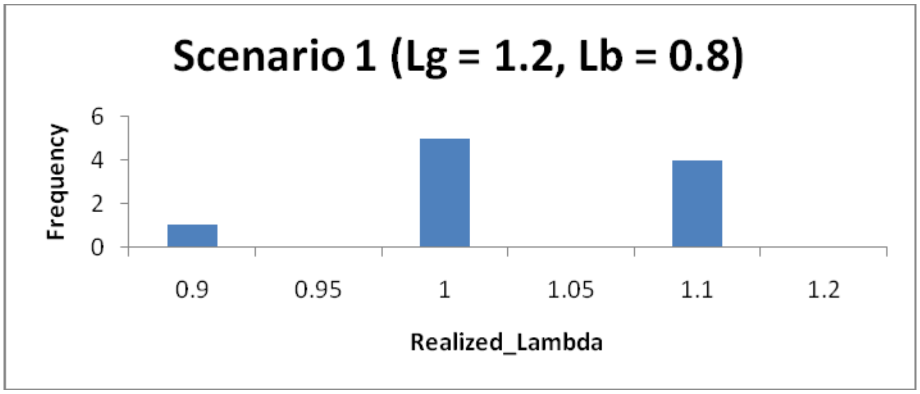
Expected_Realized_Lambda = [(1.2) * (0.8) * (1.2) * (0.8)] ^ 1/4 = 0.979

Expected_Population_Size = 1000 * 0.979 *0.979 * 0.979 * 0.979 = 922

- report the following statistics for the population sizes at time 5: mean, SD, CV, min, max.

	Pop Size N(5)
mean	936.960
min	614.400
max	1382.400
std	335.350
cv	35.791

- frequency distribution (histogram) of the realized population growth rates (lambda)



- report the following statistics for the realized population growth: mean, SD, CV, min, max.

	Realized Lambda
mean	0.973
min	0.885
max	1.084
std	0.087
cv	8.933

2 b) Repeat this entire process with the same probabilities (p_good = p_bad = 0.5), but now use different growth rates for "good" years (Lambda = 1.5) and "bad" years (Lambda = 0.5). Enter the new lambdas in the excel sheet and report the same statistics and figures.

When we change Lambda_good = 1.5 and Lambda_bad = 0.5, we expect two results:

- decreased mean realized_lambda and mean population size at N(5). When you calculate the expected realized_Lambda with the new parameters, you obtain a smaller value than for scenario 1.

  Given p(good) = p (bad) = 0.5, we would expect the population to grow at

  Expected_Realized_Lambda = [(1.5) * (0.5) * (1.5) * (0.5)] ^ 1/4 = 0.866

Expected_Population_Size = 1000 * 0.866 *0.866 * 0.866 * 0.866 = 562

- The distribution of realized_Lambda and N(5) is more variable in scenario 2 because the populations grow or decrease faster in this scenario with a larger Lambda_good and a smaller Lambda_bad. Check the CVs for the two examples and it would be larger in scenario 2. The distributions will thus show a larger range of values (higher maximums, lower minimums) for scenario 2 than for scenario 1.

	N(5)	Realized Lambda
mean	**1112.500**	**0.906**
min	**62.500**	**0.500**
max	**5062.500**	**1.500**
std	**1501.388**	**0.290**
cv	**134.956**	**31.998**

2 c) Finally, assume that now good and bad years alternate each other for ever. What would be the realized growth rate (lambda) for the four scenarios you have calculated above: (Lambda for a good year = 1.75, 1.5, 1.25, 1)? Hint: You do not need to make all of these calculations, use algebra to get the right answer. Show me your work.

Lambda_good	Lambda_bad	Calculation	Realized_Lambda
1	1	= (1*1*1*1) ^ (¼)	1.000
1.25	0.75	= (1.25*0.75*1.25*0.75) ^ (¼)	0.968
1.5	0.5	= (1.5*0.5*1.5*0.5) ^ (¼)	0.866
1.75	0.25	= (1.75*0.25*1.75*0.25) ^ (¼)	0.661

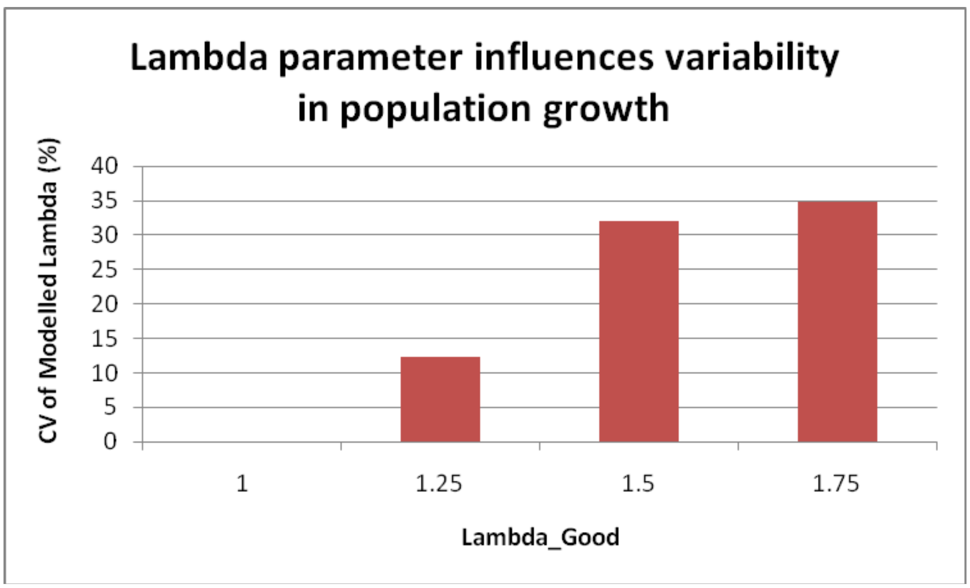
2 d) Create two plots of the "modeled" mean and the CV of the realized population growth (y axis) as a function of the "calculated" realized lambda values from question 2c (x axis). (Hint: enter the new lambda values into the excel sheet and copy and paste the output to get the realized lambda values for the missing scenarios). Discuss how the "calculated" and "modeled" mean realized population growth estimates compare to each other. What is happening as the environmental variability increases in this scenario?

Note: the Lambda = 1 scenario is trivial. The population does not grow or decrease.

For the other scenarios, as Lambda_good increases from 1 to 1.75 (and Lambda_bad decreases accordingly from 1 to 0.25), we see that the modeled mean lambda values decline, accordingly. Depending on the specific stochastic results of your runs, the modeled point estimates will be above or below the calculated expectations. However, the result is not consistent: sometimes the model results are on average better or worse… it depends on the scenario.

However, what is a consistent result is the increase in the variability of the model results with the increase in Lambda_good. This is evident in the larger values of the modeled Lambda_CV, as we change the Lambda_good parameters from 1 to 1.75.

	Modelled		Calculated
Lambda_good	Lambda_mean	Lambda_cv	Lambda_realized
1	1	0	1.000
1.25	0.9627	12.312	0.968
1.5	0.905	31.997	0.866
1.75	0.734	34.882	0.661



The reason for the smaller realized_Lambda as we increase Lambda_good and decrease Lambda_bad may seem counterintuitive, since the arithmetic means of the pairs of possible lambda values are always 1: (1 & 1), (1.25 and 0.75), (1.5 & 0.5), (1.75 & 0.25). However, the key is that population growth in this model is a multiplicative process - not an additive process – so, we need to calculate the geometric mean, not the arithmetic mean. To check that you get it, can you calculate how large Lambda_bad would have to be in this example to maintain a realized_Lambda of 1, as Lambda_good increases from 1 to 1.75? (hint: imagine we alternate from good and bad years sequentially for ever… like in this example).

The reason for the larger CV is that the population undergoes larger proportional consecutive increases / decreases, as we increase Lambda_good and decrease Lambda_bad.