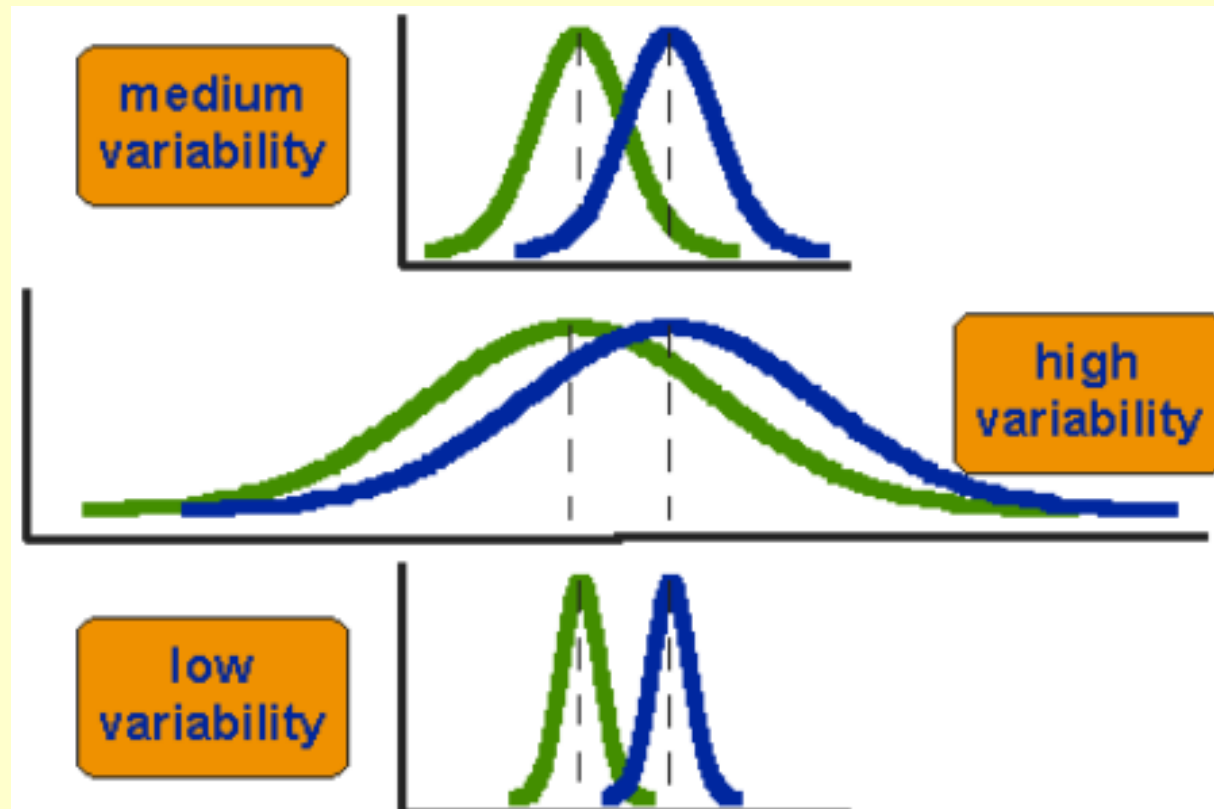


Comparing Two Means



http://www.pelagicos.net/classes_biometry_fa17.htm

T-test

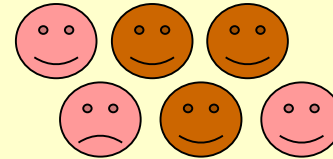
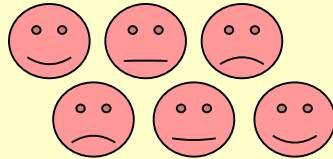
- Independent t -test (or just plain T-test)
 - Compares two means from independent observations - values are not connected.
(e.g., data from different groups of people).
- Dependent t -test (or paired T-test)
 - Compares two means based on related observations - they are not independent.
(e.g., data from same individuals).

Note: These data are from paired samples.

First Approach- Independent Samples

1. Randomly assign individuals to one treatment:

**Group
1**



**Group
2**

2. Measure the Response Variable:

Standardized metrics
Same audience

Lecturing
Skills

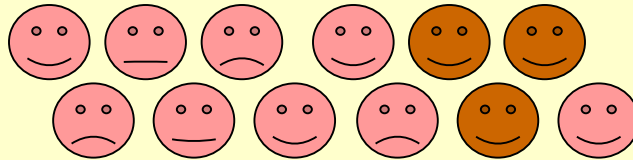
3. Compare Response Variable in Two Groups:

6 measurements (group 1)

6 measurements (group 2)

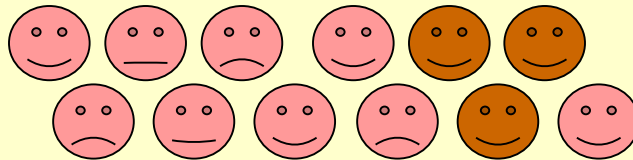
Second Approach- Dependent Samples

1. Each individual is measured (control) :



Lecturing
Skills

2. Each individual is manipulated (experiment):



Lecturing
Skills

3. Compare Response Variable in Each Individual:
12 individual measurements (control / manipulation)

Rationale for the T-test

- Two samples collected and means calculated.
- If samples come from the same population, we expect their means to be roughly equal.
- While these means may differ by chance alone, large differences occur infrequently.
- Compare the observed difference between the sample means that we collected and the difference between the sample means we would expect if there were no effect (i.e., if the null hypothesis were true).

Rationale for the T-test

- We use the standard error to gauge the variability between sample means. If the difference between the samples is larger than what we would expect based on the standard error then we can assume one of two:
 - There is no effect and we have, by chance, collected two samples that are atypical of the origin population (type I error)
 - The two samples come from two different populations. The difference between samples represents a genuine difference (reject the incorrect null hypothesis)

Rationale for the T-test

- As the observed difference between the sample means gets larger, we are more confident that second explanation is correct (i.e., the null hypothesis should be rejected).
- In an experiment, if we reject the null hypothesis, we infer that the two sample means differ because of the different experimental manipulation we imposed on each sample.
- **NOTE:** This requires random assignment of individuals to ensure there are no biases.

Generic T-test Statistic

$$t = \frac{\text{observed difference between sample means} - \text{expected difference between population means (if null hypothesis is true)}}{\text{estimate of the standard error of the difference between two sample means}}$$

$$t = \frac{\text{Observed (Mean1 - Mean 2)} - \text{Expected (Mean1 - Mean 2)}}{\text{Estimated Standard Error of the Difference (Mean1 - Mean 2)}}$$

Assumptions of the T-test

- The independent and dependent t-tests are *parametric tests* based on normal distributions.
- Therefore, they assume:
 - Observations are randomly sampled.
 - Data are on the interval or ratio scale .
 - Sampling distribution is normally distributed.

NOTE: In the dependent t test, the sampling distribution *of the differences* between scores is normal, not the scores themselves.

Assumptions of the T-test

- Because the independent t -test compares different groups of observations, it assumes:
 - Variances in these populations are equal (*homogeneity of sample variances*).
 - Scores in different treatment conditions are independent (because they come from different individuals - not paired observations).

Dependent (Paired) T-test

$$t = \frac{\bar{D} - \mu_D}{s_D / \sqrt{N}}$$

$$t = \frac{\begin{array}{c} \text{Observed} \\ \text{Mean} \\ \text{Difference} \end{array} - \begin{array}{c} \text{Expected} \\ \text{Mean} \\ \text{Difference} \end{array}}{\text{Standard Error}}$$

Dependent (Paired) T-test

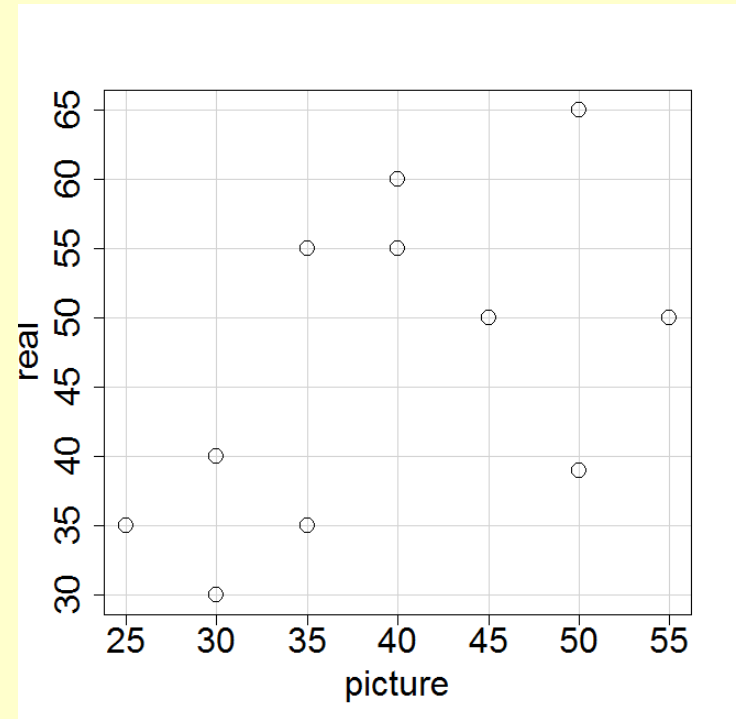
Example: Is arachnophobia specific to real spiders or is seeing a picture enough?

- Participants: 12 spider phobic individuals
- Manipulation: Participants exposed to two treatments: (at two points in time): a real spider and a picture of that same spider
- Outcome: Anxiety (Heart rate)

Why use a Paired T-test ?

We expect individuals to not be identical in their baseline level of fear (there is variability in their fear of spiders)

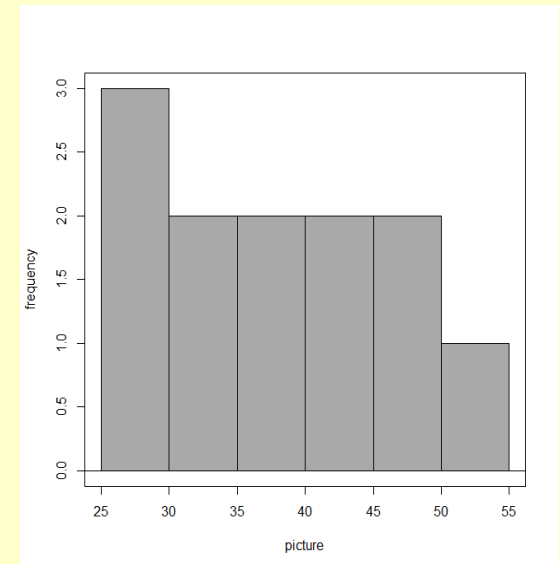
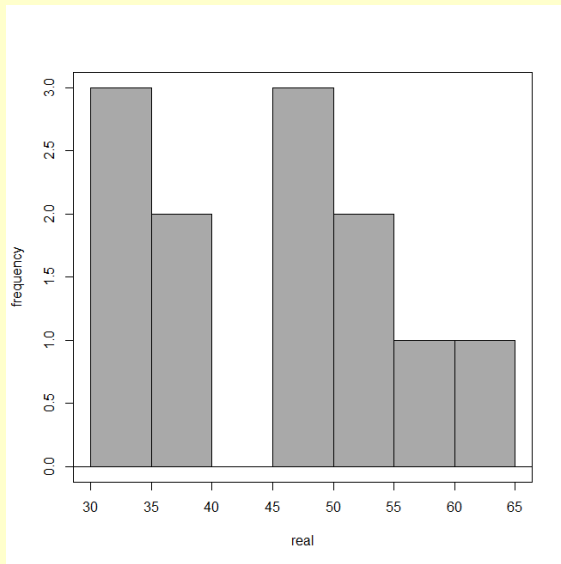
- How can we determine if, in fact some participants are more afraid than other participants, whether they see a real spider of a photo of a spider?



Pearson's correlation data:
picture and real
 $t = 2.0582$, $df = 10$,
 $p\text{-value} = 0.06659$
sample estimates:
cor 0.5454881

Paired T-test

SpiderWide.xlsx



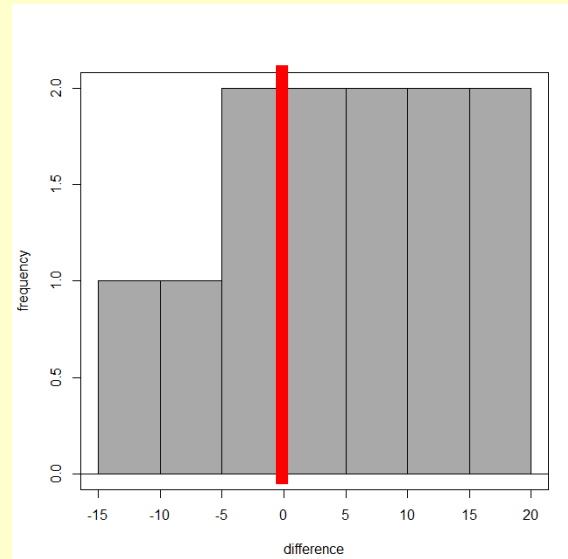
A screenshot of an R console window titled 'paired.spiders'. It displays a table with columns 'picture', 'real', and 'difference'. The data is as follows:

	picture	real	difference
1	30	40	10
2	35	35	0
3	45	50	5
4	40	55	15
5	50	65	15
6	35	55	20
7	55	50	-5
8	25	35	10
9	30	30	0
10	45	50	5
11	40	60	20
12	50	39	-11

(Real - Photo)

NOTE:

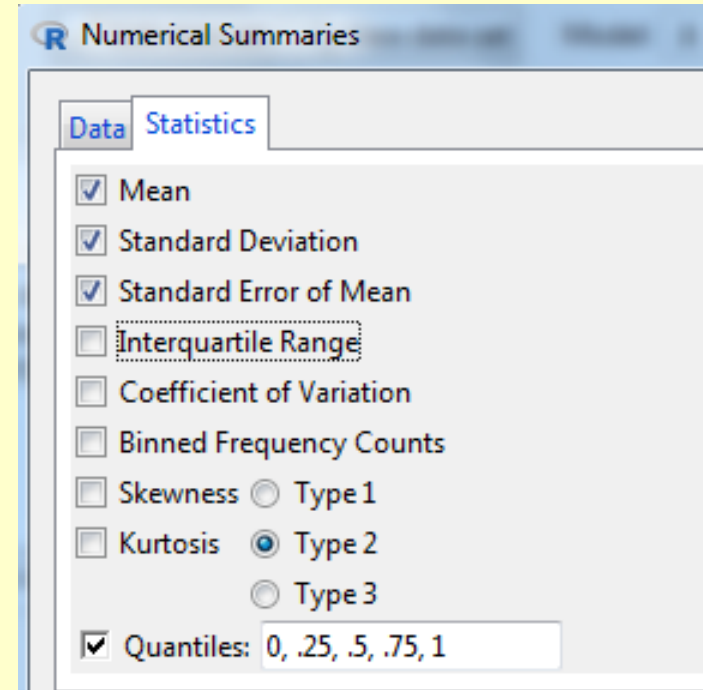
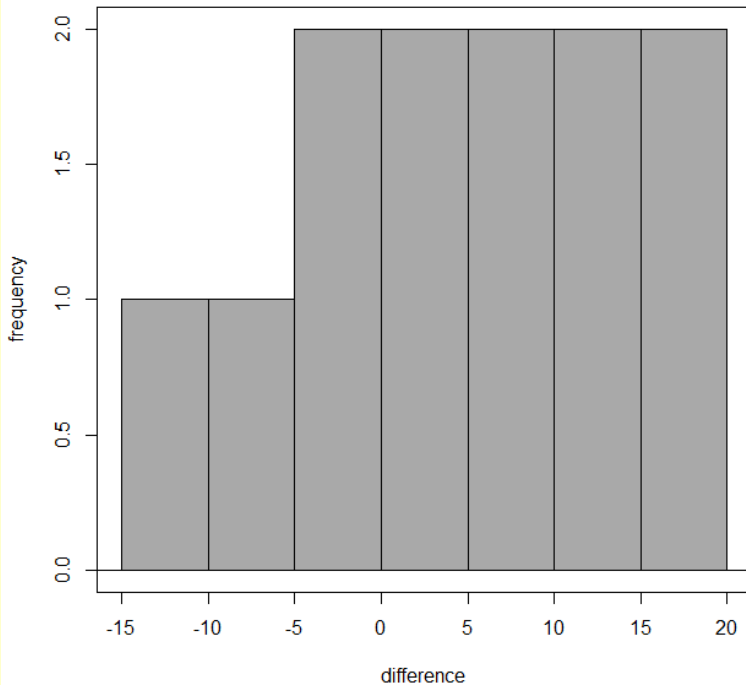
What result do you expect if there is no difference between "real" and "photo" ?



The null hypothesis states that: mean of real = mean of photo

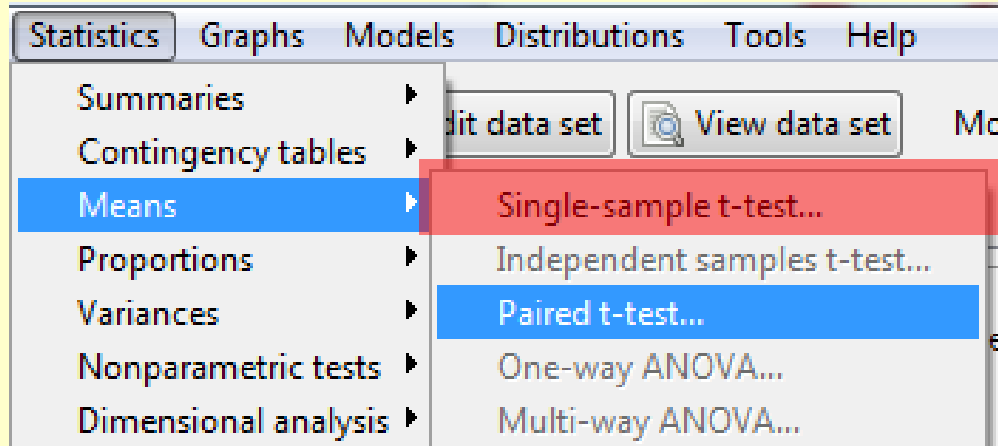
Paired T-test

(Real - Photo)



mean	sd	se(mean)	0%	25%	50%	75%	100%	n
7	9.807233	2.831104	-11	0	7.5	15	20	12

Performing a T-test in Rcmdr



Compare sample mean against theoretical mean

Independent t-test

Paired t-test
(Dependent t-test)

Independent data need to be organized this way:

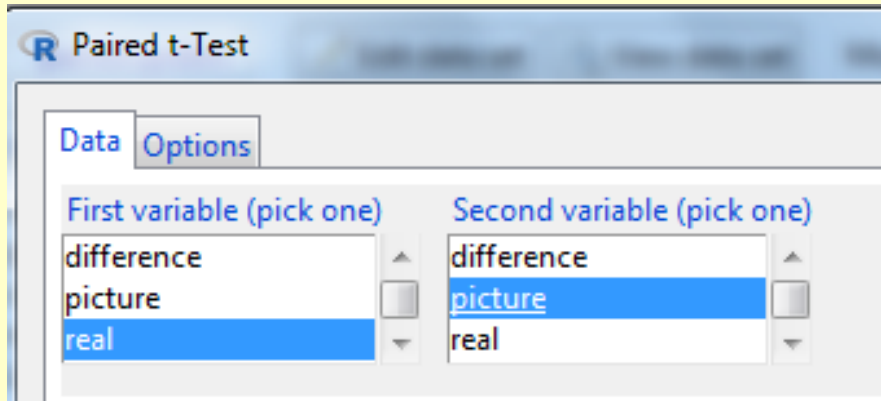
- 1 column: data
- 1 column: groups

The screenshot shows an R data frame window titled 'spiders'. It contains a single column named 'Anxiety' with 10 rows of data. All rows have the value 'Picture' in the first column and numerical values in the second column.

	Group	Anxiety
1	Picture	30
2	Picture	35
3	Picture	45
4	Picture	40
5	Picture	50
6	Picture	35
7	Picture	55
8	Picture	25
9	Picture	30
10	Picture	45

NOTE: Because there are no defined "groups", Rcmdr does not allow you to do an independent t-test

Dependent T-test Output



When performing the test, make sure you remember which one you select as first and the second variable

```
> with(paired.spiders, (t.test(real, picture,  
alternative='two.sided', conf.level=.95, paired=TRUE)))
```

Paired t-test data: real and picture

t = 2.4725, df = 11, p-value = 0.03098

alternative hypothesis:

difference in means is not equal to 0

95 percent confidence interval: 0.7687815 13.2312185

sample estimates: mean of the differences 7

Calculating the Effect Size

$$r = \sqrt{\frac{t^2}{t^2 + df}}$$

t = t statistic
(difference of means)

df = degrees of freedom

Numerator:

$$2.4725 * 2.4725 = 6.1132$$

Denominator:

$$6.1132 + 11 = 17.1132$$

$$r = \text{sqrt}(6.1132 / 17.1132)$$

$$r = 0.5977$$

If $t = 0$

(no difference)

Then $r \sim 0$

If $t = \infty$

(large difference)

Then $r \sim 1$

Reporting Dependent T-test Results

On average, participants experienced significantly greater anxiety to real spiders ($M = 47.00$, $SE = 3.18$) than to pictures of spiders ($M = 40.00$, $SE = 2.68$), as revealed by a paired t-test ($t = 2.47$, $df = 11$, $p = 0.031$).

$$t = (+7 - 0) / 2.831 = +2.471$$

$$t = (-7 - 0) / 2.831 = -2.471$$

mean	sd	se(mean)
7	9.807233	2.831104

Paired t-test data:
real and picture
 $t = 2.4725$, $df = 11$,
 $p\text{-value} = 0.03098$

NOTE: Sign of t test statistic does not matter in the two tailed test. However, it does matter for the one tailed test.

Independent (Paired) T-test

Example: Is arachnophobia specific to real spiders or is seeing a picture enough?

- Participants: 24 spider phobic individuals
- Manipulation: Participants exposed to one of two treatments: 12 to a real spider and a 12 to a picture of that same spider
- Outcome: Anxiety (Heart rate)

Independent T-test

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$

Observed Mean 1 - Observed Mean 2

Sqrt (Pooled Sample Variance)

Sample Pooled Variance:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

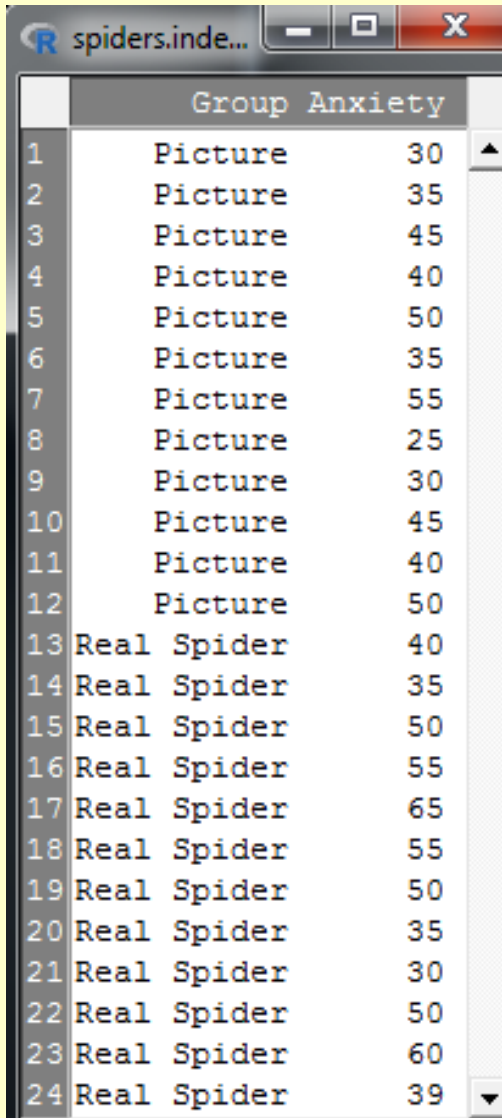
Independent (Unpaired) T-test

Example: Is arachnophobia specific to real spiders or is seeing a picture enough?

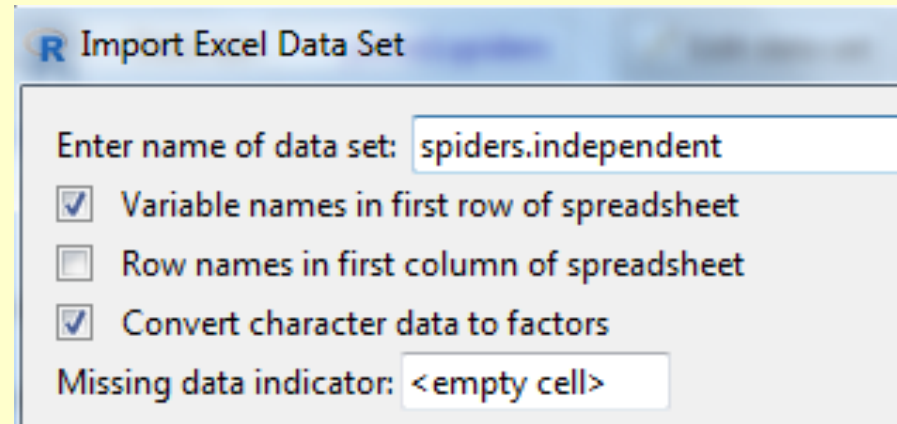
- Participants: 24 spider phobic individuals
- Manipulation: Participants exposed to one of two treatments: 12 to a real spider and a 12 to a picture of that same spider
- Outcome: Anxiety (Heart rate)

Independent T-test

Groups:
Picture
Real



	Group	Anxiety
1	Picture	30
2	Picture	35
3	Picture	45
4	Picture	40
5	Picture	50
6	Picture	35
7	Picture	55
8	Picture	25
9	Picture	30
10	Picture	45
11	Picture	40
12	Picture	50
13	Real Spider	40
14	Real Spider	35
15	Real Spider	50
16	Real Spider	55
17	Real Spider	65
18	Real Spider	55
19	Real Spider	50
20	Real Spider	35
21	Real Spider	30
22	Real Spider	50
23	Real Spider	60
24	Real Spider	39

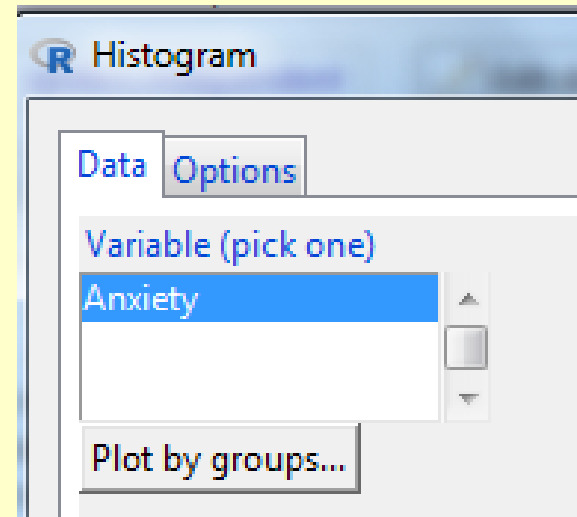


Import Excel Data Set

Enter name of data set: spiders.independent

- Variable names in first row of spreadsheet
- Row names in first column of spreadsheet
- Convert character data to factors

Missing data indicator: <empty cell>



Histogram

Data Options

Variable (pick one)

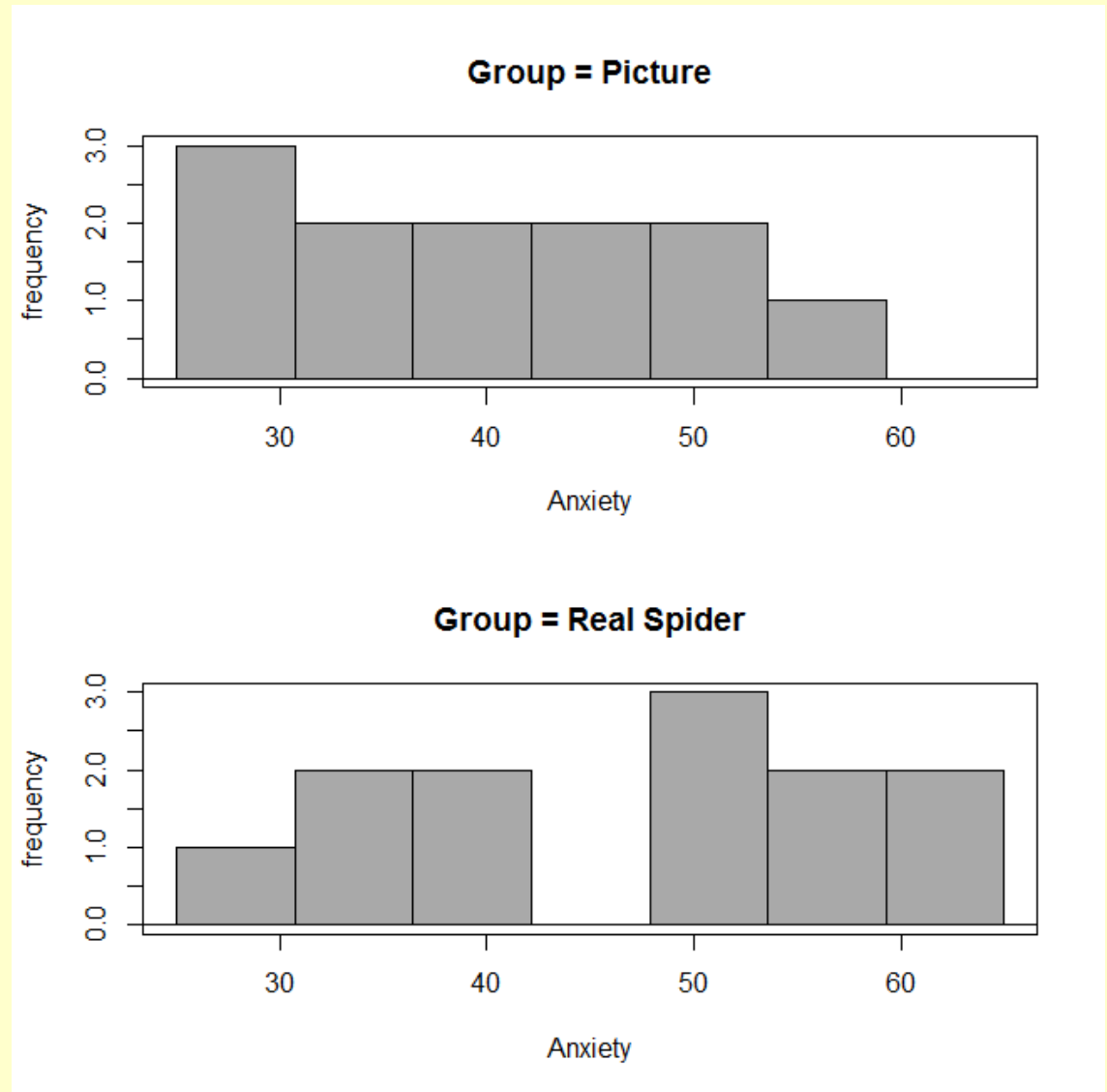
Anxiety

Plot by groups...

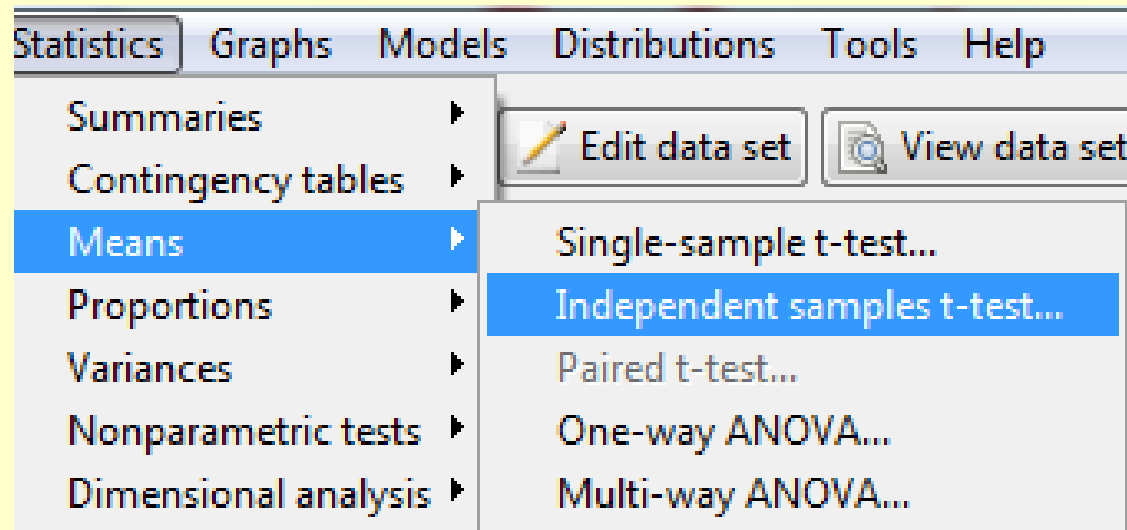
Independent T-test

SpiderLong.xlsx

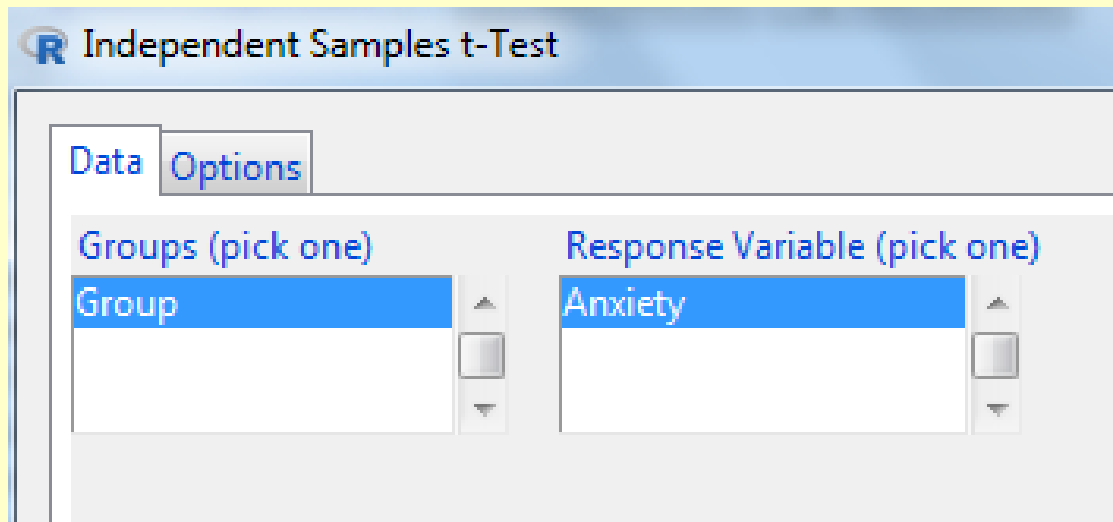
	Group	Anxiety
1	Picture	30
2	Picture	35
3	Picture	45
4	Picture	40
5	Picture	50
6	Picture	35
7	Picture	55
8	Picture	25
9	Picture	30
10	Picture	45
11	Picture	40
12	Picture	50
13	Real Spider	40
14	Real Spider	35
15	Real Spider	50
16	Real Spider	55
17	Real Spider	65
18	Real Spider	55
19	Real Spider	50
20	Real Spider	35
21	Real Spider	30
22	Real Spider	50
23	Real Spider	60
24	Real Spider	39



Performing a T-test on SPSS

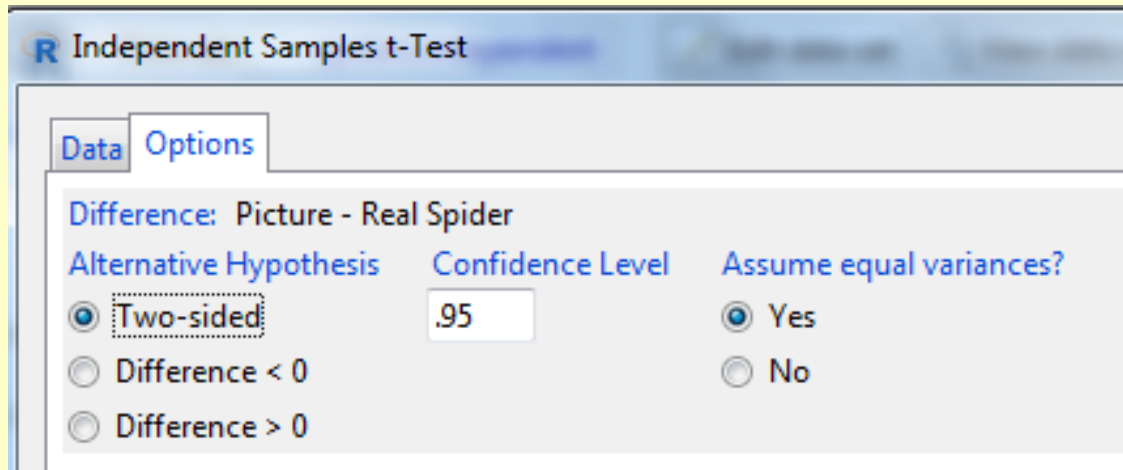


Independent t-test



- Select Groups
- Select Response Variable

Independent T-test Output



Assume equal
Variances

(After testing
with Levene's or
Bartlett's Test)

```
> t.test(Anxiety~Group, alternative='two.sided',  
conf.level=.95, var.equal=TRUE, data=spiders.independent)
```

Two Sample t-test data:

Anxiety by Group $t = -1.681$, $df = 22$, $p\text{-value} = 0.1068$

alternative hypothesis:

true difference in means is not equal to 0

95 percent confidence interval: -15.634222 1.634222

sample estimates:

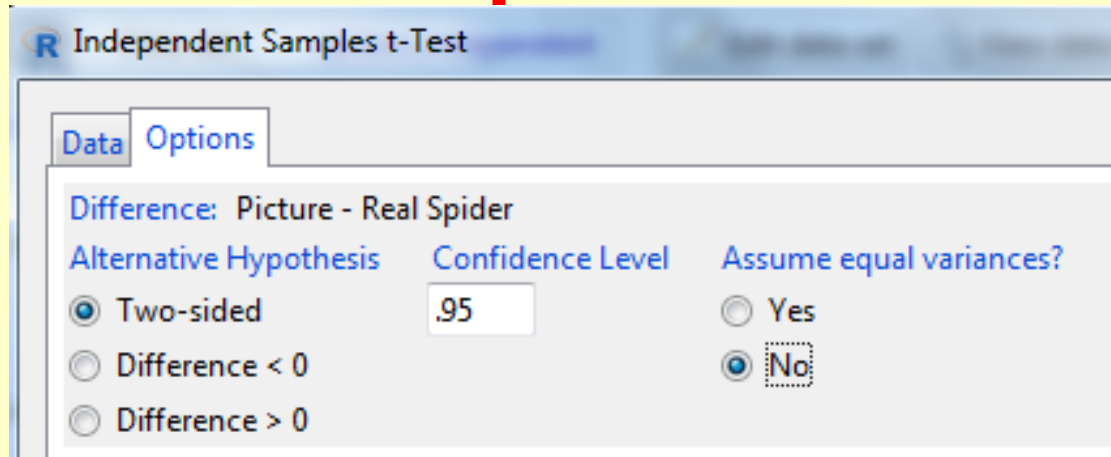
mean in group Picture

40

mean in group Real Spider

47

Independent T-test Output



Do not Assume equal variances (After testing with Levene's or Bartlett's Test)

```
> t.test(Anxiety~Group, alternative='two.sided',  
conf.level=.95, Rcmdr+ var.equal=FALSE,  
data=spiders.independent)
```

Welch Two Sample t-test data:

Anxiety by Group $t = -1.681$, $df = 21.385$, $p\text{-value} = 0.107$
alternative hypothesis: difference in means not equal to 0
95 percent confidence interval: -15.648641 1.648641 sample
estimates:

mean in group Picture
40

mean in group Real Spider
47

Calculating the Effect Size

$$r = \sqrt{\frac{t^2}{t^2 + df}}$$

t = t statistic
(difference of means)

df = degrees of freedom

Numerator:

$$-1.681 * -1.681 = 2.8257$$

Denominator:

$$2.8257 + 22 = 24.8257$$

$$r = \text{sqrt}(2.8257 / 24.8257)$$

$$r = 0.3373$$

If $t = 0$
(no difference)
Then $r \sim 0$

If $t = \infty$
(large difference)
Then $r \sim 1$

Reporting Dependent T-test Results

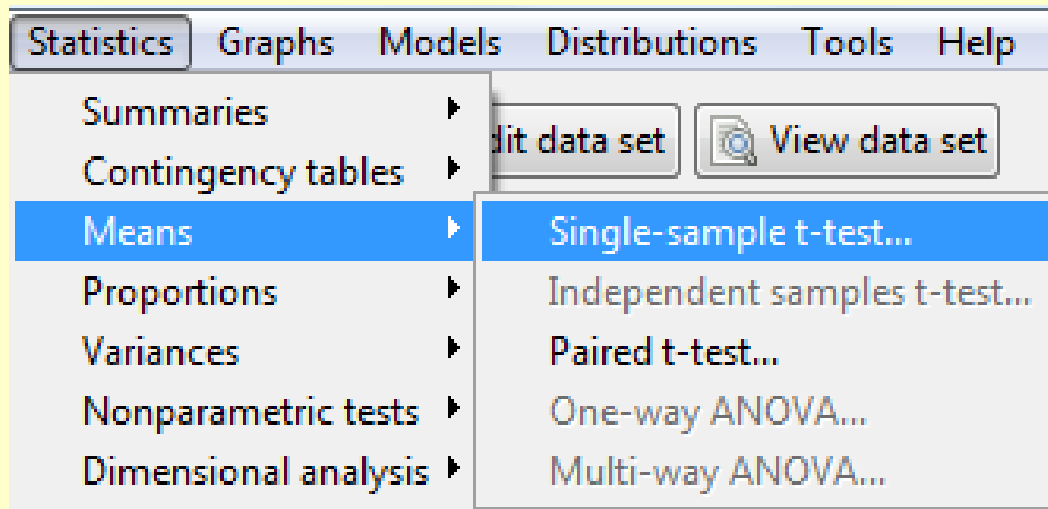
On average, participants experienced greater anxiety to real spiders ($M = 47.00$, $SE = 3.18$), than to pictures of spiders ($M = 40.00$, $SE = 2.68$). While a t- test did not reveal a statistically significant difference ($t = -1.68$, $df = 22$, $p = 0.107$); had a medium-sized effect ($r = 0.34$).

$$t = (+7 - 0) / 4.163 = +1.681$$

$$t = (-7 - 0) / 4.163 = -1.681$$

NOTE: Sign of t test statistic does not matter in the two tailed test. However, it does matter for the one tailed test.

The One-Sample T-test in Rcmdr

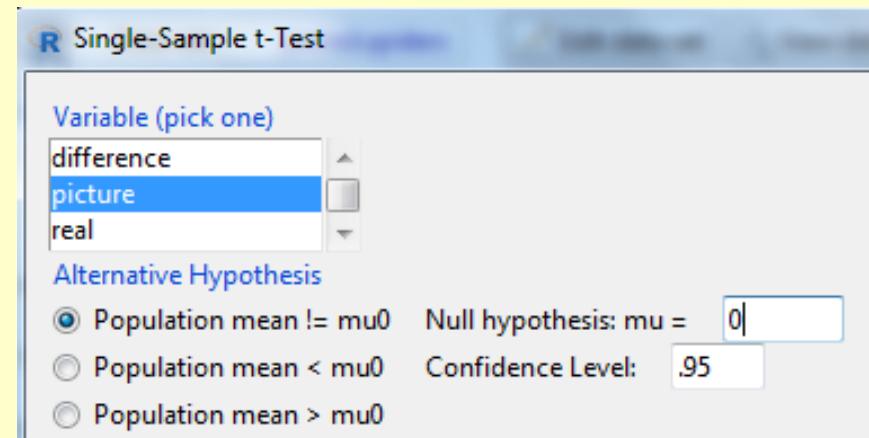


Compare sample mean against theoretical mean

Use dependent data and test each group:

	picture	real
1	30	40
2	35	35
3	45	50
4	40	55
5	50	65
6	35	55
7	55	50
8	25	35
9	30	30
10	45	50
11	40	60
12	50	39

Select variable and null hypothesis mean (μ_0):



The One-Sample T-test in Rcmdr

```
> with(paired.spiders, (t.test(picture,  
alternative='two.sided', mu=0, Rcmdr+ conf.level=.95)))
```

One Sample t-test data:

```
picture t = 14.91, df = 11, p-value = 0.00000001214  
alternative hypothesis: true mean is not equal to 0  
95 percent confidence interval: 34.09538 45.90462  
sample estimates: mean of x 40
```

```
> with(paired.spiders, (t.test(picture,  
alternative='two.sided', mu=40, conf.level=.95)))
```

One Sample t-test data:

```
picture t = 0, df = 11, p-value = 1  
alternative hypothesis: true mean is not equal to 40  
95 percent confidence interval: 34.09538 45.90462  
sample estimates: mean of x 40
```

T-test Summary

- T-tests allow us to analyze simple experiments involving only one independent variable manipulated in two ways and only one measured outcome variable.
- T-tests can be independent or dependent (paired); on the basis of whether the same individuals are measured one or two times.
- Independent T-tests should allocate individuals to treatments randomly to avoid individual biases.
- Whenever possible, use paired tests because they allow you to control for individual-level variation.