between the warm waters of the Sargasso Sea and the cooler waters over the continental margin. Armed with an understanding of geostrophic currents we can see that the sharp gradient in temperature (and hence density) across the Stream is an expression of a balance between the horizontal pressure gradient force and the Coriolis force, an equilibrium that has been brought about indirectly by the wind. (We will return to geostrophic equilibrium in the Gulf Stream in Section 4.3.2.)

4.2 THE SUBTROPICAL GYRES

As discussed in Section 3.4, current flow in the subtropical gyres is related to the overlying anticyclonic wind systems, which blow around the subtropical high pressure regions. The centres of the atmospheric and oceanic gyres are not, however, coincident: the centres of the atmospheric gyres tend to be displaced towards the eastern side of the oceans, while the centres of the oceanic gyres tend to be displaced towards the western side, especially in the Northern Hemisphere. As a result, the currents that flow along the western sides of oceans—the **western boundary currents**—are characteristically fast, intense, deep and narrow, while those that flow along the eastern sides—the **eastern boundary currents**—are characteristically slow, wide, shallow and diffuse.

The western boundary current in the North Atlantic is the Gulf Stream; its counterpart in the North Pacific is the Kuroshio. Both of these currents are typically only some 100km wide and in places have surface velocities in excess of $2ms^{-1}$. By contrast, the Canary Current and the California Current are over 1000km wide and generally have surface velocities less than $0.25ms^{-1}$. In the South Atlantic and the South Pacific, the difference between the western boundary currents (the Brazil Current and the East Australian Current, respectively) and the eastern boundary currents (the Benguela Current and the Humboldt, or Peru, Current) is not so marked. This may be because the South Atlantic and South Pacific are open to the Southern Ocean so that their gyres are strongly influenced by the Antarctic Circumpolar Current; furthermore, the South Pacific does not have a continuous barrier along its western boundary. However, some of the differences between the gyres of the Northern and Southern Hemispheres may be apparent rather than real. The northern gyres have been more fully investigated; and perhaps when more information is available, the southern gyres will be seen to resemble their northern counterparts more closely.

4.2.1 VORTICITY

> 'Big whirls have little whirls
> which feed on their velocity.
> Little whirls have lesser whirls
> and so on to viscosity.'
>
> L. F. Richardson (1881–1953), referring to atmospheric motions and paraphrasing Augustus de Morgan (who had himself paraphrased Jonathan Swift).

You will already be familiar with the idea—essential to an understanding of dynamic systems—that energy and mass must be conserved. Another

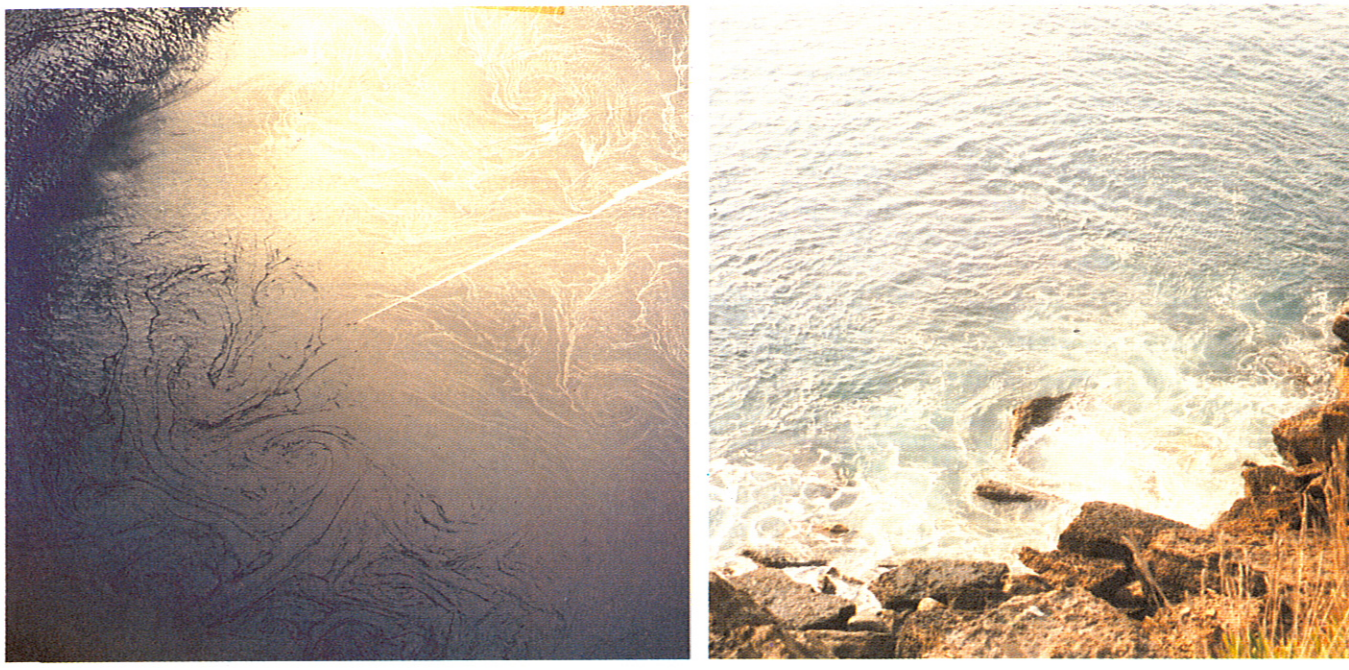


Figure 4.4 Examples of eddies in the oceans. *Left:* Eddies in the Mediterranean off the coast of Libya. They were photographed from the Space Shuttle and show up because of variations in surface roughness. The picture shows an area about 75km across; the white line is a ship's wake or bilge dump. *Right:* Small-scale eddies forming in the vicinity of a rocky coastline.

property that must be conserved is momentum—both linear momentum associated with motion in straight lines and angular momentum associated with rotatory movement. In oceanography, it is more convenient to view the conservation of angular momentum as the conservation of a tendency to rotate, or the conservation of **vorticity**, the tendency to form vortices.

Ocean waters have rotatory motions on all scales, from the basin-wide subtropical gyres down to the smallest swirls and eddies (Figure 4.4). Fluid motion does not have to be in closed loops to be rotatory; whenever there is **current shear** (a change in velocity at right angles to the direction of flow) there is a tendency to rotate and the water has vorticity. For convenience, *a tendency to rotate anticlockwise is referred to as positive and a tendency to rotate clockwise is referred to as negative*. These aspects of vorticity are illustrated schematically in Figure 4.5.

Note that we say 'a *tendency* to rotate' rather than simply 'rotatory motion'. This is because water may acquire positive vorticity by one mechanism at the same time as it acquires negative vorticity by another. For example, water may be acquiring positive vorticity as a result of

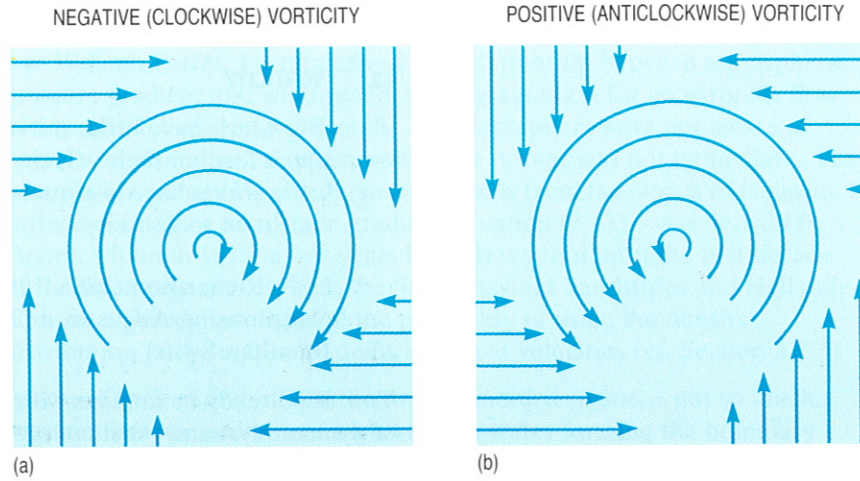


Figure 4.5 Diagrams to show examples of flow with (a) negative and (b) positive vorticity. The speed and direction of the flow are indicated by the lengths and directions of the arrows.

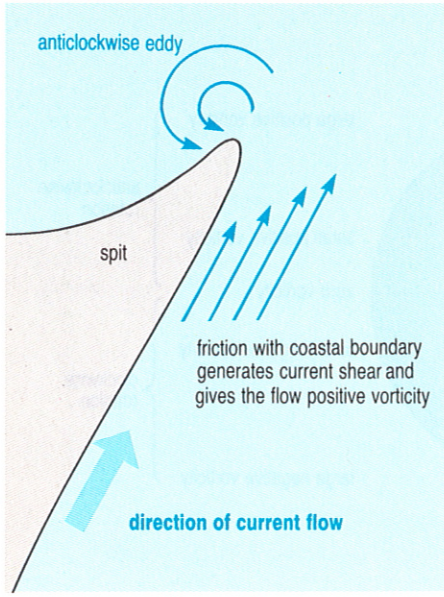


Figure 4.6 Diagram to illustrate how eddies are generated off spits; the lengths and directions of the arrows indicate the speed and direction of the current.

current shear caused by friction with a coastal boundary such as a spit (Figure 4.6) or adjacent water, while at the same time it is acquiring negative vorticity from the wind. The actual rotatory motion that results will depend on the relative sizes of the two effects. In theory, positive and negative vorticity tendencies could be exactly equal so that *no* rotatory motion would result.

Water that has a rotatory motion in relation to the surface of the Earth, caused by wind stress and/or frictional forces, is said to possess **relative vorticity**. However, the Earth itself is rotating. The vorticity possessed by a parcel of fluid by reason of its being on the rotating Earth is known as its **planetary vorticity**.

Planetary vorticity and the Coriolis force

In Chapter 1, you saw how the rotation of the Earth about its axis results in the deflection of currents and winds by the Coriolis force. These deflections were explained in terms of the poleward decrease in the eastwards velocity of the surface of the Earth. In addition to a linear eastwards velocity, the surface of the Earth has an *angular velocity*. Like the linear eastwards velocity, this angular velocity depends on latitude. At the poles, the angular velocity of the surface of the Earth is simply Ω ($= 7.29 \times 10^{-5} s^{-1}$ (*cf.* Section 3.2)). At lower latitudes, it is a proportion of Ω: the bigger the angle between the Earth's axis of rotation and the local vertical axis, the smaller the angular velocity of the surface of the Earth about this vertical axis. At the Equator, where a vertical axis is at right angles to the axis of rotation of the Earth, the angular velocity of the surface is zero. In general, the angular velocity of the surface of the Earth about a vertical axis at latitude ϕ is given by $\Omega \sin \phi$ (Figure 4.7(a)).

You saw in Chapter 1 that a missile fired northwards from the Equator is deflected eastwards in relation to the surface of the Earth because, with increasing latitude, the surface of the Earth travels eastwards at a progressively decreasing rate. However, as just described, the surface of the Earth also has an *angular* velocity, so that in the Northern Hemisphere it rotates anticlockwise about a vertical axis, and in the Southern Hemisphere it rotates clockwise (Figure 4.7(b)). For this reason, there would be relative motion between a hypothetical missile and the surface of the Earth, regardless of the direction in which the missile is fired (as long as it is not fired along the Equator), and this relative motion would increase with increasing latitude. Similarly, winds and currents moving eastwards and westwards experience deflection, as well as those moving northwards and southwards. You have seen that the Coriolis force causing this deflection is weakest in the tropics and strongest at the poles. Using the fact that the angular velocity of the surface of the Earth is given by $\Omega \sin \phi$, it is possible to deduce the exact relationship of the Coriolis force to latitude, which you have already encountered in Section 3.1.2:

$$\begin{aligned} \text{Coriolis force} &= m \times 2\Omega \sin \phi \times u \\ &= mfu \end{aligned} \qquad \text{(equation 3.2)}$$

where m is the mass of a body moving with velocity u, and f ($= 2\,\Omega \sin \phi$) is known as the Coriolis parameter.

Clearly, if the surface of the Earth at latitude ϕ has an angular velocity of $\Omega \sin \phi$, so too does any object or parcel of fluid on the Earth at that latitude. In other words, any parcel of fluid on the surface of the Earth

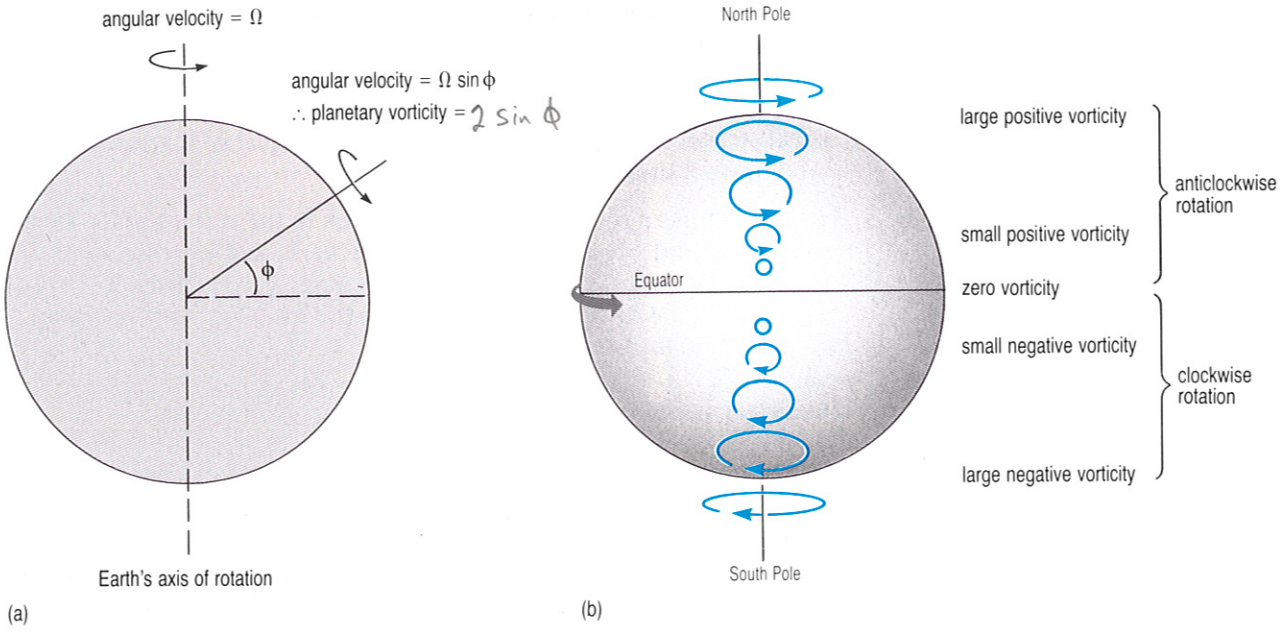


Figure 4.7 (a) Diagram to show the derivation of the expression for the planetary vorticity of a fluid parcel on the surface of the Earth at latitude ϕ. (For use with Question 4.3(a).)

(b) Schematic diagram to show the variation of planetary vorticity with latitude; the blue arrows represent the component of the Earth's rotation about a vertical axis (viewed from above) at different latitudes.

shares the component of the Earth's angular rotation appropriate to that latitude.

The vorticity of a parcel of fluid is defined mathematically to be equal to twice its angular velocity.

QUESTION 4.3 (a) What, then, is the planetary vorticity possessed by a parcel of fluid on the surface of the Earth at latitude ϕ? (To answer this Question, complete Figure 4.7(a).)

(b) What is the planetary vorticity (i) at the North Pole, (ii) at the South Pole, and (iii) at the Equator?

In answering Question 4.3(a), you will have seen that planetary vorticity is given by the same expression as the Coriolis parameter; for that reason, it also is given the symbol f. Relative vorticity is given the symbol ζ (zeta).

At the beginning of this Section it was stated that vorticity—effectively, angular momentum—must be conserved. But it is vorticity *relative to fixed space* (i.e. with respect to an observer outside the Earth) that must be conserved; in other words, it is the **absolute vorticity** of a parcel of fluid—the combination of the planetary and relative vorticity ($f + \zeta$)—that must remain constant, in the absence of external forces such as wind stress and friction. Consider, for example, a parcel of water that is being carried northwards from the Equator in a current. As it moves northwards it will be moving into regions of increasingly large positive (anticlockwise) planetary vorticity (see Figure 4.7(b)). However, as it is only weakly bound to the Earth by friction, it tends to be 'left behind' by the Earth rotating beneath it and, relative to the Earth, acquires an increasingly negative (clockwise) rotational tendency*. If there are no external influences imparting positive or negative relative vorticity to the parcel of water (e.g. it does not come under the influence of cyclonic or

*If you find this difficult to envisage, you can understand what we mean by being 'left behind' simply by filling a glass with water and twirling it. You will find that the water tends to remain stationary, as the glass rotates outside it. Of course, *in relation to the glass* the water *is* moving.

anticyclonic wind fields), the increase in positive planetary vorticity will be exactly equal to the increase in negative relative vorticity, and the absolute vorticity ($f + \zeta$) will remain constant.

QUESTION 4.4 (a) A body of water is carried *southwards* from the Equator in a current.

(i) As the water moves south, how is its planetary vorticity changing?

(ii) How does this affect its relative vorticity?

(b) What happens to the relative vorticity of a body of water if it is acted upon by:

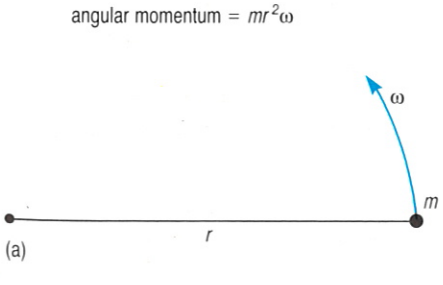
(i) winds blowing in a clockwise direction?

(ii) cyclonic winds in the Southern Hemisphere?

However, the situation is not quite as straightforward as this. In order to understand what happens in the oceans, we have to bear in mind that the vorticity of a body of water is the *sum* of the vorticity of all the constituent particles of water. Consider, for convenience, a column of water moving in a current. For the purposes of this discussion, we can imagine the column of water spinning about its own axis, although to have vorticity it could of course have any type of rotatory motion (Figure 4.5). For simplicity, we can also assume that the column is rotating anticlockwise in the Northern Hemisphere, i.e. it has positive relative vorticity.

What will happen if the rotating column of water becomes longer and thinner as a result of stretching, because it moves into a region of deeper sea-floor, for example? (Consider what happens when spinning skaters bring their arms in close to their sides.)

The simple answer is that the column of water will spin faster, i.e. its relative vorticity will increase (become more positive). Looked at from the point of view of angular momentum, the angular momentum of each particle of water is $mr^2\omega$, where r is the distance from the particle (of mass m) to the axis of rotation, and ω is the angular velocity (Figure 4.8(a)). When the column stretches, the average radius r goes down and so, for angular momentum to be conserved, ω the speed of rotation must increase (Figure 4.8(b)). Because of the effect of changes in the length



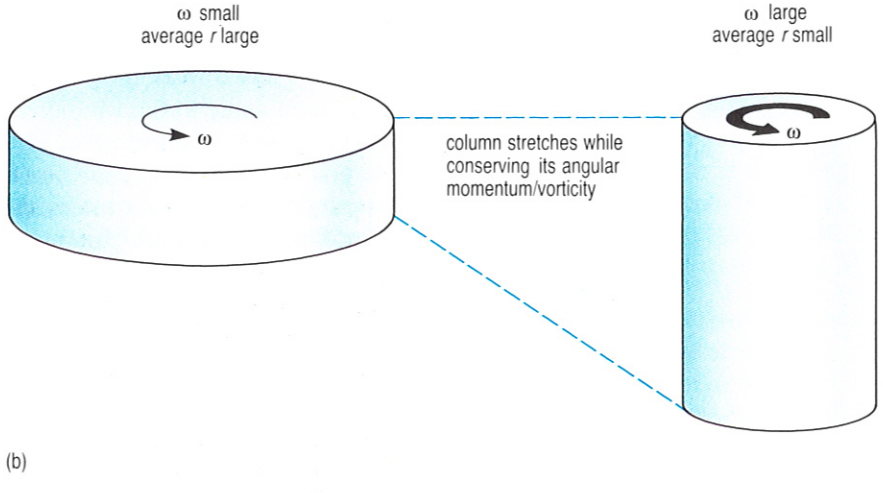


Figure 4.8 (a) The angular momentum of a particle of mass m moving with angular velocity ω in a circle of radius r is given by $mr^2\omega$.

(b) For the total angular momentum (vorticity) of a stretched column of water to remain constant, the angular velocity ω of the particles in the column must increase.

consideration of variations in f, the planetary vorticity. In fact, in the rea oceans, away from coastal boundaries and other regions of large current shear, f is very much greater than ζ. This means that $(f + \zeta)/D$ is effectively equal to f/D and it is f that must change in response to change in D; and as f is simply a function of latitude, it can only be changed by water changing its latitude.

QUESTION 4.5 (a) Imagine a current flowing from east to west in the Northern Hemisphere. What will happen if the current flows over a shallow bank, so that the depth D is reduced? (You may assume that thi occurs in the open oceans and that $f >> \zeta$.)

(b) What will happen to a zonal* current flowing over a shallow bank i the Southern Hemisphere?

The fact that conservation of f/D causes currents to swing equatorwards and polewards over topographic highs and lows, respectively, is sometimes referred to as topographic steering.

In studies of potential vorticity in the real oceans, D, the 'depth of the water column', need not be the total depth to the sea-floor. It is the thickness of the body of water under consideration and so could, for example, be the depth from the sea-surface to the bottom of the permanent thermocline or, for deep-water movements, the depth from t bottom of the permanent thermocline to the sea-floor.

Since the mid-1980s, potential vorticity has become an important tool in the study of the movement of oceanic water masses; this application of vorticity will be discussed in Chapter 6.

4.2.2 WHY IS THERE A GULF STREAM?

We will now summarize briefly, with as little mathematics as possible, some of the developments in ideas that have led to an increased understanding of the dynamics of ocean circulation in general, and of th subtropical gyres in particular. In the course of this discussion, we will g a substantial way towards answering the question posed above: why *is* there a Gulf Stream?

Ekman's initial work on wind-driven currents, intended to explain curre flow at an angle to the wind direction, was published in 1905. In 1947, th Scandinavian oceanographer H.U. Sverdrup used mathematics to demonstrate another surprising relationship between wind stress and ocean circulation. In constructing his theory, Ekman had assumed a hypothetical ocean which was not only infinitely wide and infinitely deep but also had no horizontal pressure gradients because the sea-surface an all other isobaric surfaces were assumed to be horizontal. Sverdrup's ain was to determine current flow in response to wind stress *and* horizontal

*Zonal means *either* from east to west *or* from west to east; the equivalent word for north–south flow is 'meridional'.