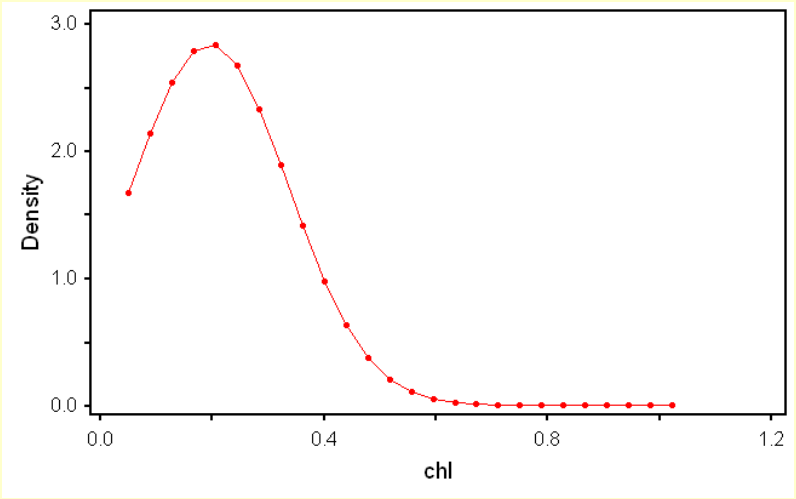
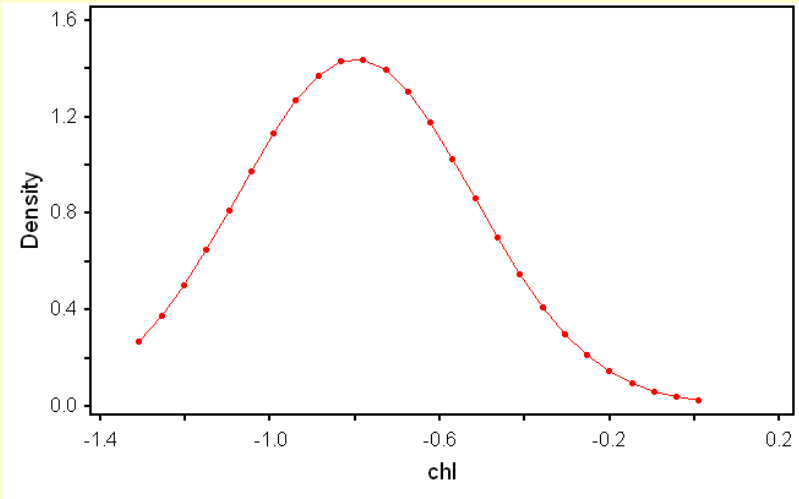
Data Transformations





Chl Concentration → Log (Chl Conc.)

http://www.pelagicos.net/classes_biometry_fa16.htm

The Theory

Normality tests assess the likelihood that the given data set $\{x_1, \ldots, x_n\}$ comes from a normal distribution.

The null hypothesis H_0 is that the observations are distributed normally with unspecified mean μ and variance σ^2, versus the alternative H_a that the data do not follow a normal distribution.

Normality can be assessed in three generic ways:

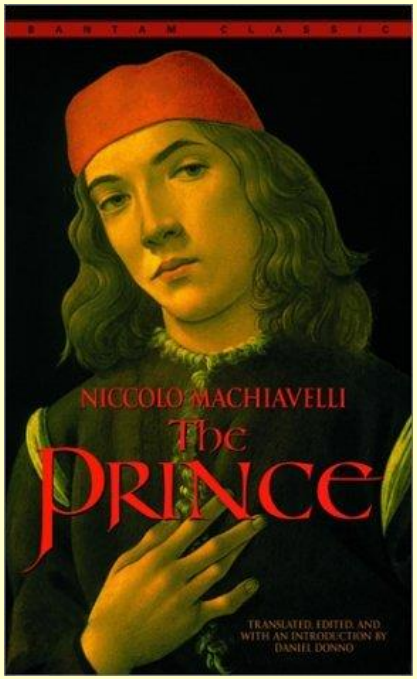
- Graphically
- Quantitatively
- Statistically

The Practice – The End Justifies the Means

Everyone wants to use parametric statistics

Many assume their data are normal

Others do not want to know



What are the options at hand:

- Before a test : transform the data

- After a test : test residuals

Approach

Powerful parametric tests are based on assumptions:

- Normality (data from a normal distribution)

  Can be assessed in three ways:

  - Graphically: P-P and Q-Q plos
  - Quantitatively: Skew & Kurtosis (Z-score)
  - Statistically: K-S and S-W Tests

- Additionally, many parametric tests require homogeneity of variances (equal variances)

  Can be assessed in three ways:

  - Graphically
  - Quantitatively
  - Statistically

Assessing Variance Homogeneity

Graphs: Scatterplots (e.g., Regressions)

Variance Ratio: (with 2 or more groups)

VR = Largest variance / Smallest variance

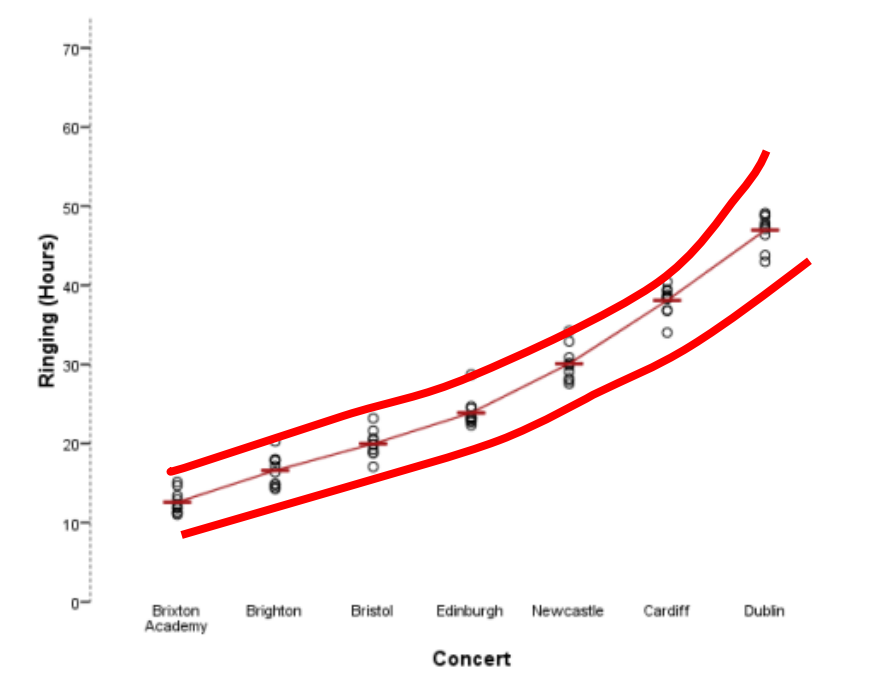
If VR < 2, can assume homogeneity

Levene's Test: Tests if variances are equal

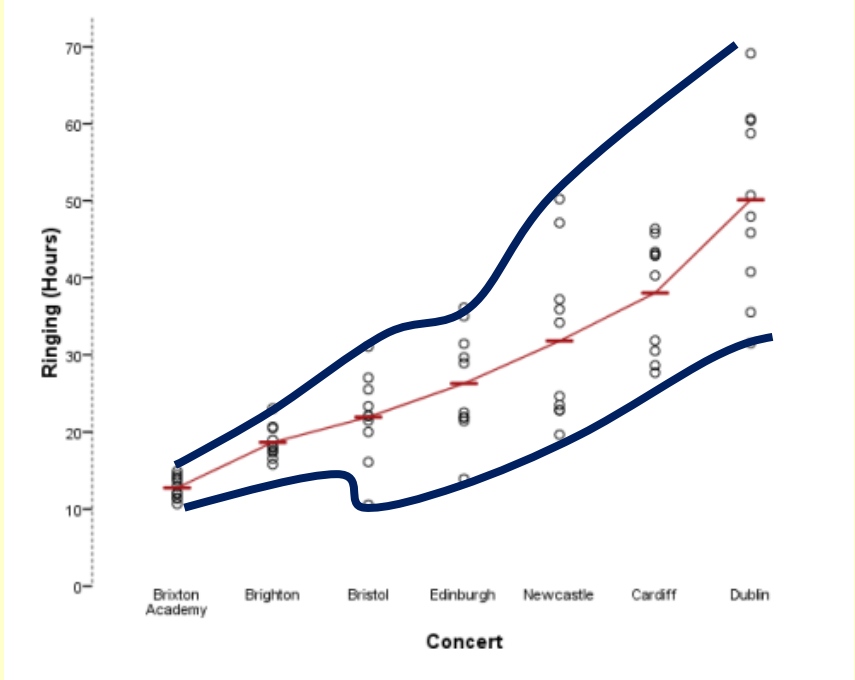
Significant = Variances are not equal

Non-Significant = Variances are equal

Variance Homogeneity - Graphic



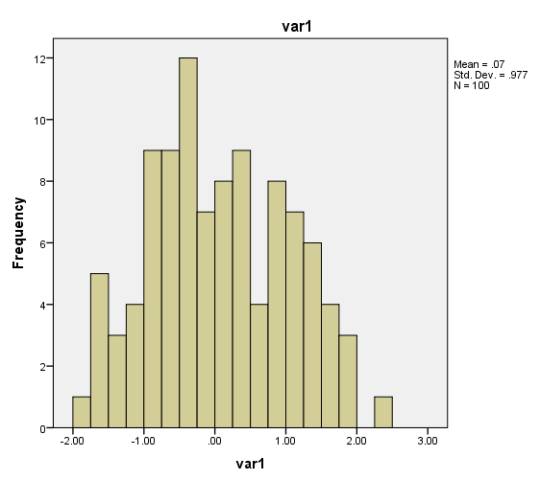
Homogeneous

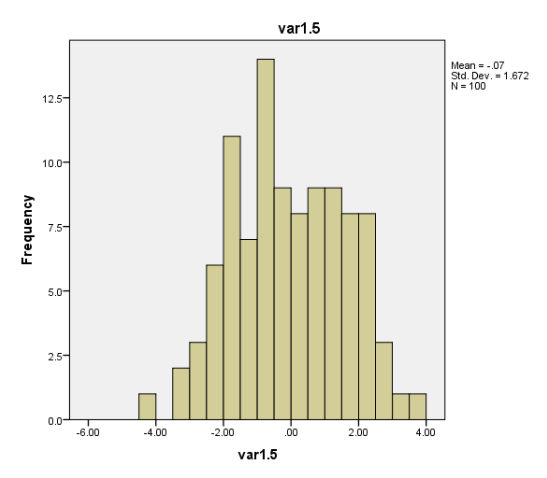


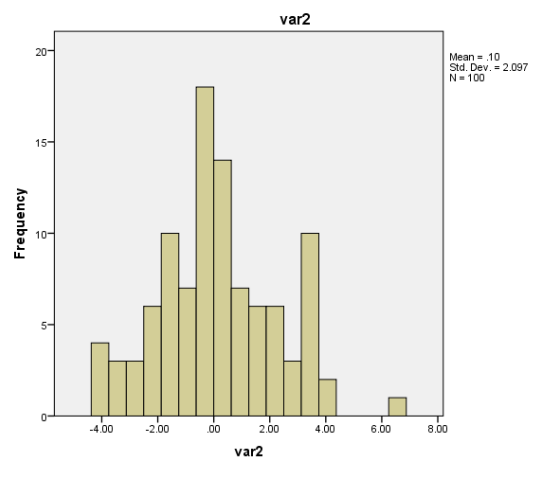
Heterogeneous

Variance Homogeneity – Ratio

Comparing three normal distributions with the same means and different variances: 1, 2.25, 4







Pairwise Variance Comparisons

Larger	Smaller	Ratio
4	1	4

Rule of Thumb:
Ratio> 2

Variance Homogeneity - Test

data	level
-.71	1.00
-.49	1.00
1.25	1.00
1.57	1.00
-.25	1.00
.68	1.00
-1.24	1.00
-.91	1.00
.78	1.00
1.46	1.00
-.25	1.00
1.19	1.00
1.28	1.00
-.83	1.00
.53	1.00
-1.47	1.00
-.80	1.00

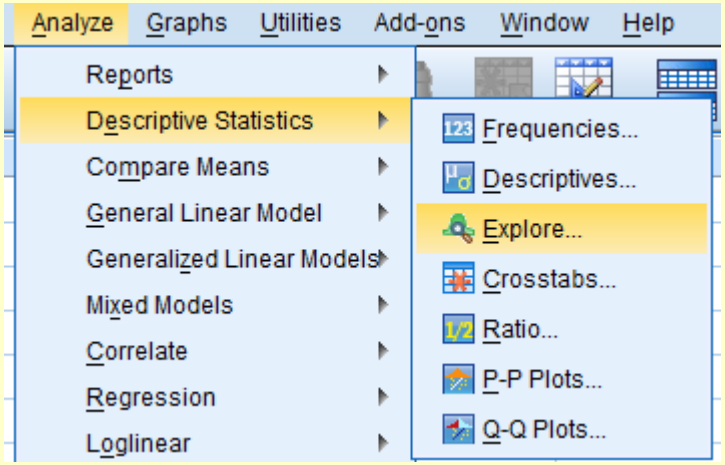
We can test all three distributions at once:

-Use one column for the data (combine all data in one column)

-Use another column for levels (of categorical factor: distribution)

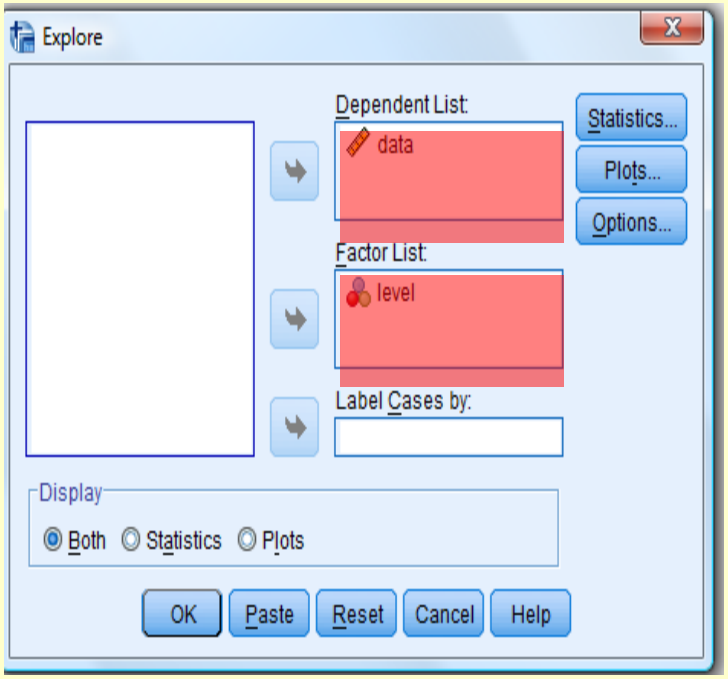
Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align	Measure
data	Numeric	8	2		None	None	8	Right	Scale
level	Numeric	8	2		None	None	8	Right	Nominal

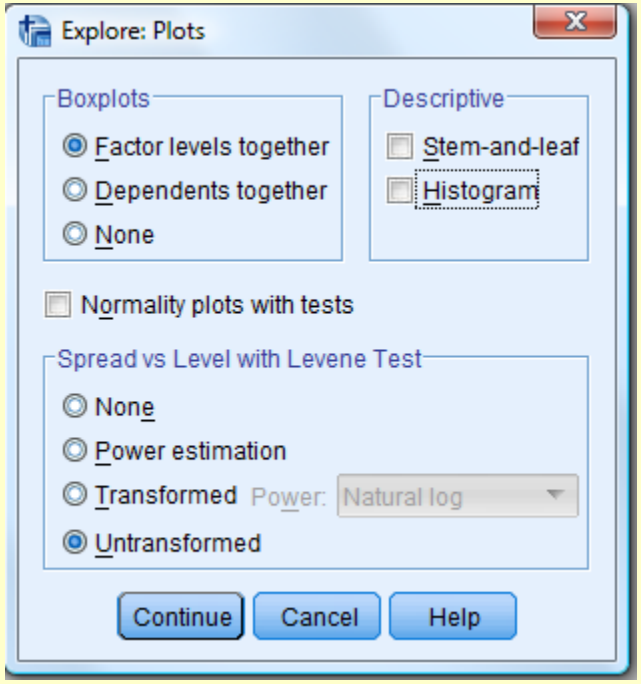
Variance Homogeneity - Test



Identify the dependent list (the data you are analyzing)

Identify the factor (group)





Boxplots

Levene's Test (No transform)

Variance Homogeneity - Test

Test of Homogeneity of Variance

		Levene Statistic	df1	df2	Sig.
data	Based on Mean	14.213	2	297	.000
	Based on Median	14.242	2	297	.000
	Based on Median and with adjusted df	14.242	2	255.942	.000
	Based on trimmed mean	14.203	2	297	.000

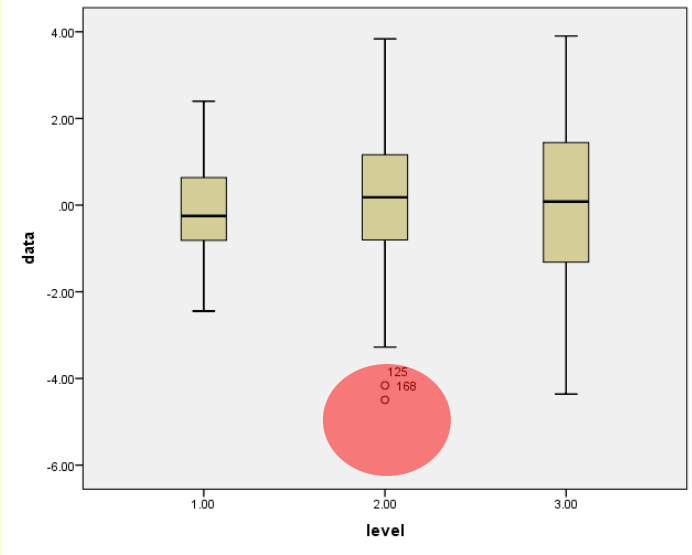
Df = 2 (k-1), 297
(Total = 300 – 1)

p < 0.001
Significant
(Different)

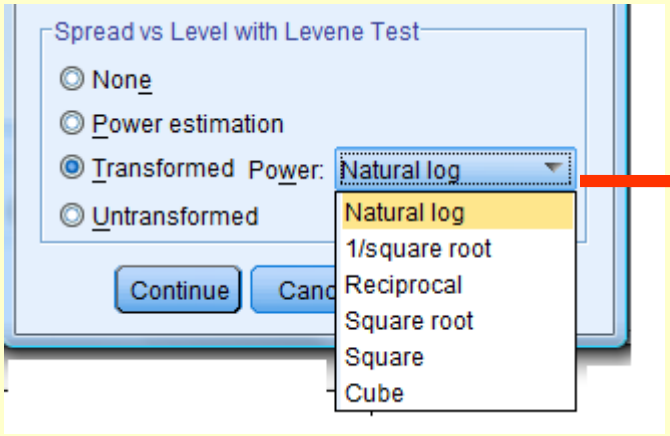
Box Plots:

For each distribution
5, 25, 50, 75, 95 %

Outliers (beyond)



Correcting Data Problems



→ Transformations

Most Important Rule: Do not Reverse the Order of the Values (larger remains larger… smaller remains smaller)

Monotonic: change values but retain ranks

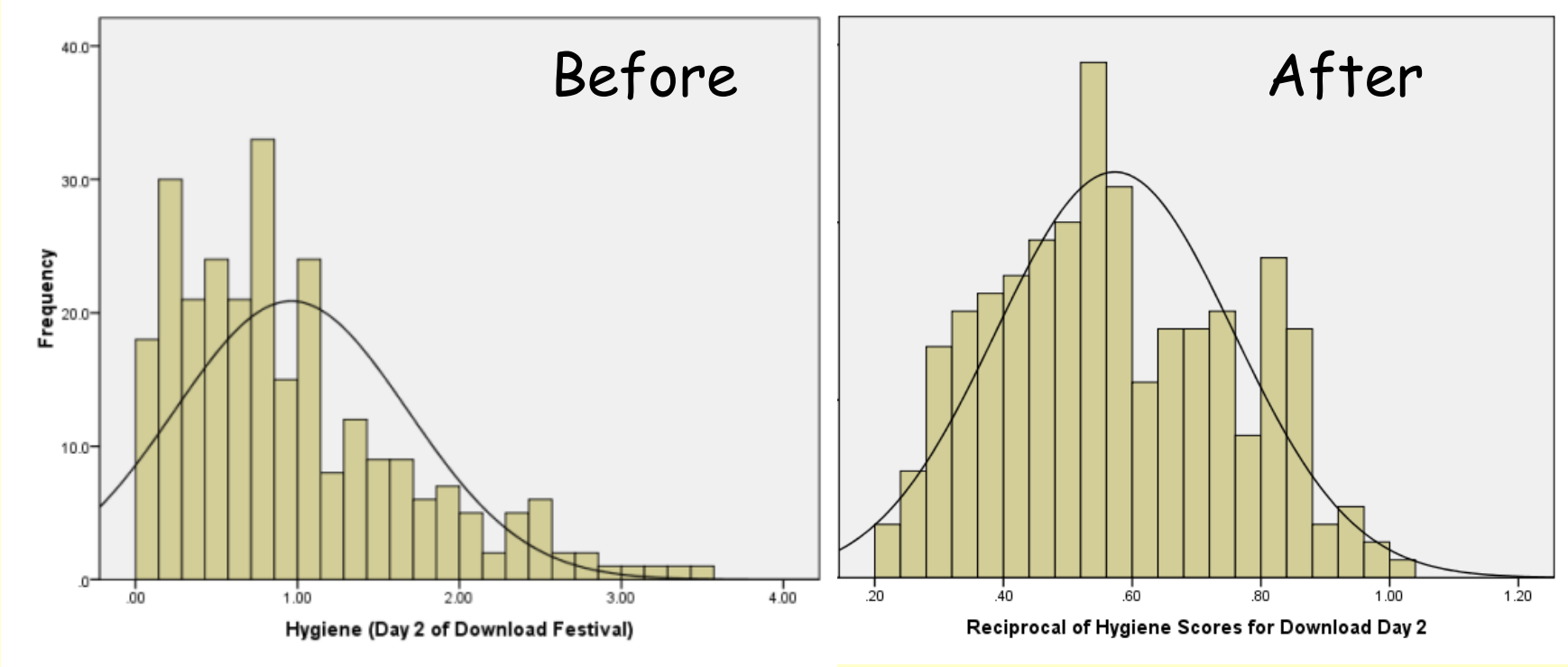
Non-monotonic: change values and ranks

(For example: Add random number, Multiply by (-1))

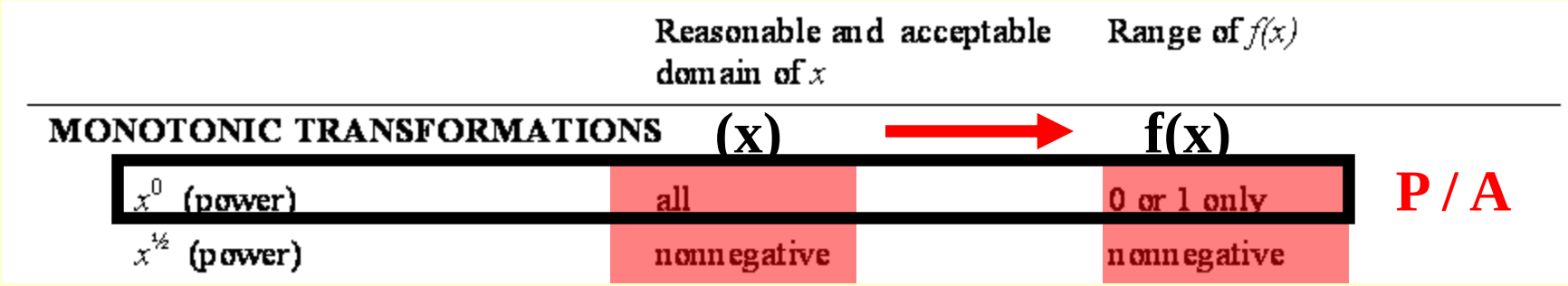
For Example: Reciprocal Transformation

$(1 / X_i)$: Dividing 1 by each value reduces the impact of large scores.

Beware: This transformation is non-monotonic, because it reverses the scores.



Monotonic Data Transformations



Power exponents:

$\frac{1}{2}$ power (square root)

Square root transform is similar, but less drastic than the log transform.
Special treatment of zeros not necessary

Note: 0 power transformation is NOT really monotonic

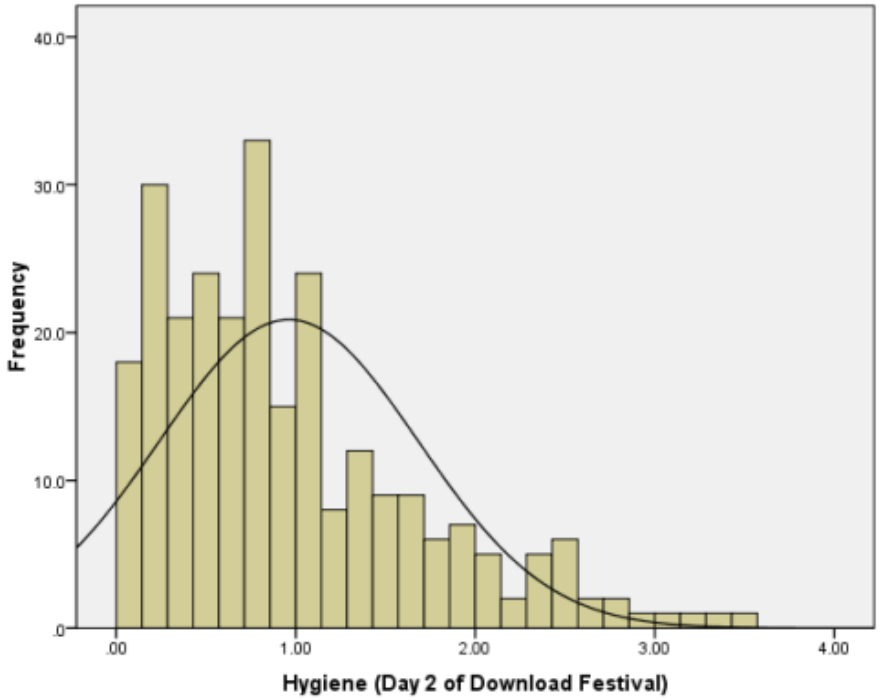
It recodes data as Presence / Absence (0 / 1)

Square Root Transformation

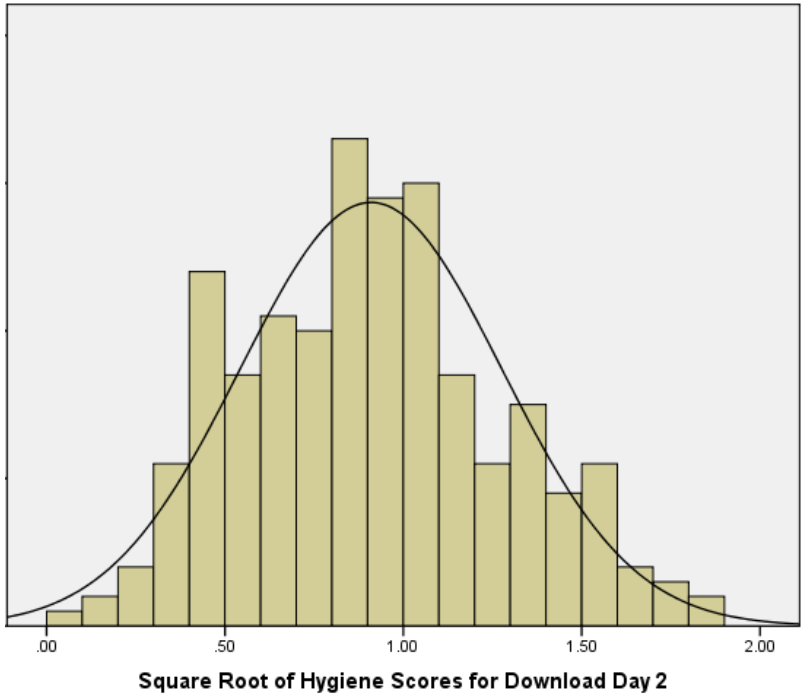
Square Root Transformation ($\sqrt{X_i}$):

Reduces positive skew. Useful for stabilizing variance

Before



After



Data Transformations – Example I

Logarithmic transformation fx = ln(x) OR log(x)

TRANSFORMATION	Reasonable and acceptable domain of x	Range of $f(x)$
$\log(x)$	positive	all

- This transformation is useful when:

- High degree of variation within samples (e.g., Chl Conc.)
- Large outliers (tails) and lots of zeros

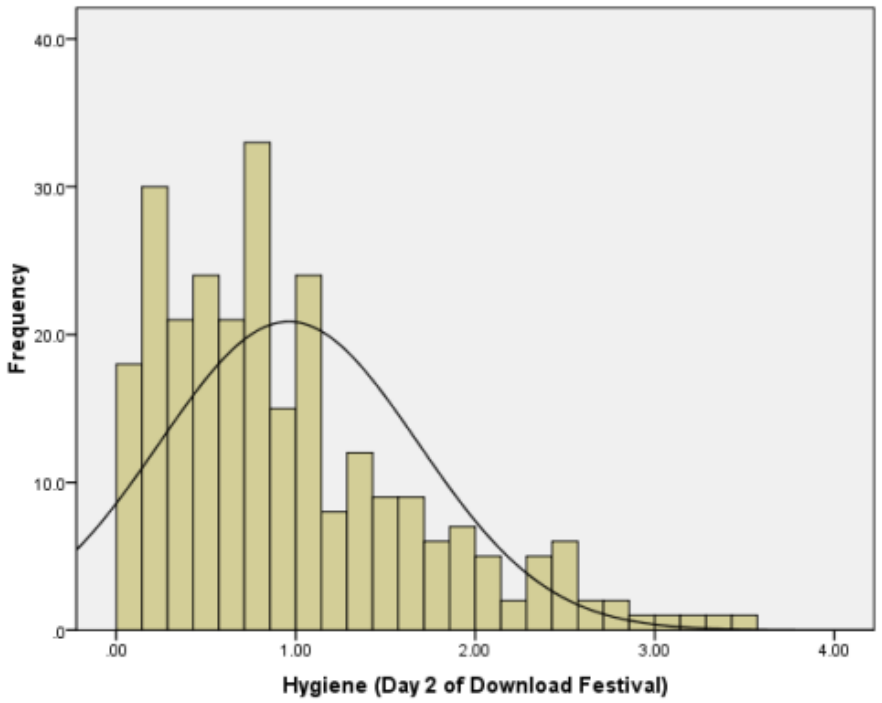
- Note: to log-transform data containing zeros, a small number should be added to all data points.

- With count data, add one, so that: fx = log(0+1) =0
- With density data, add constant smaller than smallest possible sample, so that: fx = log(0+0.001) = -3

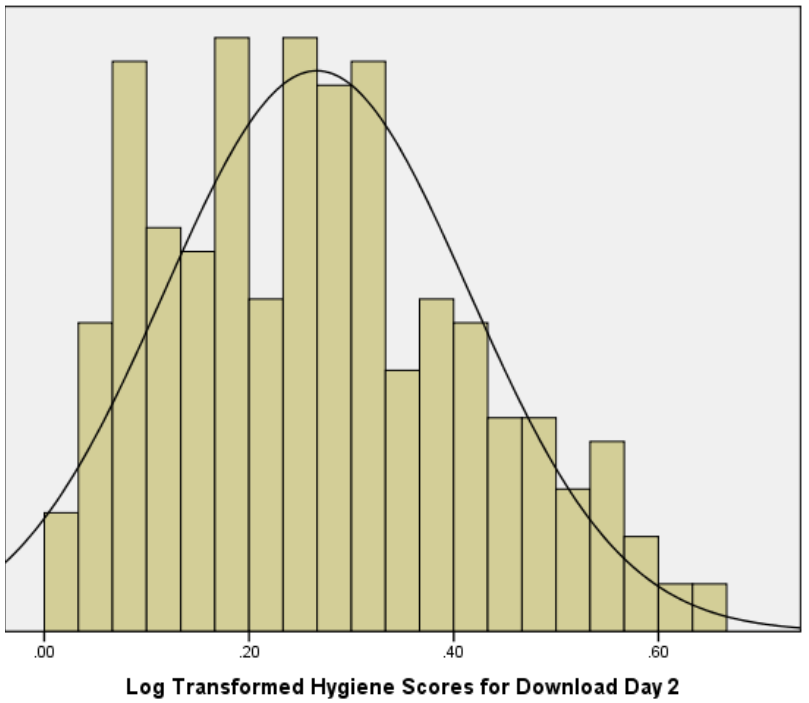
Log Transformation

Log Transformation ($\log(X_i)$): Reduces positive skew

Before



After



Data Transformations – Example II

Arcsine / Arcsine-squareroot transformation

TRANSFORMATION	Reasonable and acceptable domain of x	Range of $f(x)$
$(2/\pi)\cdot\arcsin(x)$	$0 \leq x \leq 1$	0 to 1 inclusive
$(2/\pi)\cdot\arcsin(x^{1/2})$	$0 \leq x \leq 1$	0 to 1 inclusive

- This transformation is useful when:
  - Dealing with proportional data (e.g., Percent Cover)

- Note: data must range between 0 and 1, inclusive.

The constant 2 / pi scales the result of arcsin(x) [in radians] to range from 0 to 1, assuming that $0 \leq x \leq 1$.

To Transform or Not ?

- Tricky to achieve normality with small samples
- Transforming data does not always work (e.g., fix skew / kurtosis)
- Transforming data affects hypothesis being tested
  - E.g. when using a log transformation and comparing means you switch from comparing arithmetic means to comparing geometric means

Summary

- Parametric tests based on normal distributions
- If possible, we want to use powerful parametric tests
- While significant differences can be achieved more easily with parametric statistics ...
- Critical to ensure a normal underlying data distribution, when using a parametric statistical model to avoid committing type-I error
- Remember:

  Even if the underlying distribution is normal, normality requires a large enough sample size ($n > 30$)

Homework #4

Practice Testing for Normality:

Open the file BIOL4090_Hw4_data.xls with SPSS and use these observations, drawn from three random variable distributions (derived from theoretical distributions with mean = 10 and variance =10) for the following exercise.

Make sure variables are "numeric" and measure is "scale".

Create a frequency table of each dataset of 100 data points each and use this information to fill in the table below.